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Frank J. Fabozzi

WITH STEVEN V. MANN

CHAPTER
FORTY-SEVEN

BOND IMMUNIZATION: AN ASSET/LIABILITY OPTIMIZATION STRATEGY

FRANK J. FABOZZI, PH.D., CFA, CPA
Frederick Frank Adjunct Professor of Finance
School of Management
Yale University

The purpose of this chapter is to review the mechanics and applications of the bond immunization strategy. In the first section we define immunization as a duration-matching strategy, and then compare it with maturity-matching as an alternative approach to locking in rates. To hedge the reinvestment risk present in maturity-matching, we then explain the single-period immunization strategy and the rebalancing procedures that accompany it. Following single-period immunization, we discuss multiperiod immunization and its applications for the pension, insurance, and thrift markets. Finally, we review variations on the strategy, including combination matching, contingent immunization, immunization with futures, and immunization with options.

WHAT IS AN IMMUNIZED PORTFOLIO?

Single-period immunization is usually defined as locking in a fixed rate of return over a prespecified horizon, such as locking in a 10% return for a five-year period. It also can be defined as generating a minimum future value at the end of a specified horizon, such as generating $100 million from a $70 million investment five years earlier. With multiperiod immunization, the horizon over which rates are locked in is extended to include multiple periods (such as a schedule of monthly payouts to retirees of a pension plan). Multiperiod immunization is a duration-matching strategy that permits funding of a fixed schedule of multiple future payouts at a minimum cost (such as funding a $500 million schedule of payouts at a cost of $200 million).

Previous versions of this chapter appearing in prior editions of the *Handbook* were coauthored with Peter Christensen and Anthony LoFaso.

The actuary generally credited with pioneering the immunization strategy, F. M. Reddington, defined immunization in 1952 as "the investment of the assets in such a way that the existing business is immune to a general change in the rate of interest."[1] He also specified a condition for immunization: The average duration of assets must be set equal to the average duration of the liabilities. He thought that by matching the durations of assets and liabilities he would then immunize a portfolio from the effects of small changes in interest rates. By matching durations on both sides of the balance sheet, he felt that assets and liabilities would be equally price-sensitive to changes in the general level of interest rates. For any change in yield, both sides of the ledger should be equally affected; therefore, the relative values of assets and liabilities would not be changed.

Much later, Lawrence Fisher and Roman Weil defined an immunized portfolio as follows:[2]

> A portfolio of investments is immunized for a holding period if its value at the end of the holding period, regardless of the course of rates during the holding period, must be at least as large as it would have been had the interest rate function been constant throughout the holding period.
>
> If the realized return on an investment in bonds is sure to be at least as large as the appropriately computed yield to the horizon, then that investment is immunized.

Fisher and Weil demonstrated that to achieve the immunized result, the average duration of the bond portfolio must be set equal to the remaining time in the planning horizon, and the market value of assets must be greater than or equal to the present value of the liabilities discounted at the internal rate of return of the portfolio.

Before reviewing the logic of this portfolio strategy, let's look at maturity-matching as an early approach to locking in a current level of interest rates.

MATURITY-MATCHING: THE REINVESTMENT PROBLEM

Suppose that an investor wishes to lock in prevailing interest rates for a 10-year period. Should she buy 10-year bonds?

By purchasing 10-year bonds and holding them to maturity, an investor can be certain of receiving all coupon payments over the 10-year period, as well as the principal repayment at redemption (assuming that no default occurs). These two sources of income are fixed in dollar amounts. The third and final source of income is the interest earned on the semiannual coupon payments. "Interest on coupon" is not fixed in dollar amounts; rather, it depends on the many interest-rate environments at the various times of payment.

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1. F. M. Reddington, "Review of the Principle of Life-Office Valuations," *Journal of the Institute of Actuaries* 78(1952), pp. 286–340.
2. Lawrence Fisher and Roman Weil, "Coping with the Risk of Interest-Rate Fluctuations: Returns to Bondholders from Naive and Optimal Strategies," *Journal of Business* (October 1971), pp. 408–431.

A reinvestment problem occurs when the reinvestment of coupon income occurs at rates below the yield-to-maturity of the bond at the time of purchase. Note from Exhibit 47–1 that as interest rates shift instantaneously and remain at the new levels for a 10-year period, the total "holding period" return on a 9% par bond due in 10 years will vary considerably. The initial effect will appear in the value of the asset. The immediate result will be a capital gain if rates fall (or loss, if rates rise).

As the holding period increases after a change in rates, the interest-on-coupon component of total return begins to exert a stronger influence. At 10 years, we note that interest on coupon (reinvestment income) exerts a dominance over capital gain (or loss) in determining holding-period returns.

Intuitively, we know that these relationships make sense. Capital gains appear instantly, whereas changes in reinvestment rates take time to exert their effect on the total holding-period return on a bond.

If rates were to jump immediately from 9% to 15% and a capital loss were to appear today, at what point will that capital loss be made up because the reinvestment of coupon payments is occurring at a higher (15%) rate? As illustrated in Exhibit 47–2, the two "offsetting forces" of market value and reinvestment return equally offset at 6.79 years. This is the duration of the 10-year, 9% bond. To earn the original 9% target return (the yield-to-maturity at the time of purchase), it is necessary to hold that bond for the period of its duration—6.79 in our example. If we wish to lock in a market rate of 9% for a 10-year period, we would select a bond with a duration of 10 (not a maturity of 10 years). The maturity for such a par bond in a 9% yield environment is roughly 23 years.

From Exhibit 47–1, we note that regardless of the immediate, one-time interest-rate shift, we are still able to earn a 9% total return if our holding period is 6.79 years—the duration of the bond. By targeting the duration of a portfolio rather than specific maturities to the prescribed investment horizon of 6.79, we see the equal offsets of capital gain with lower reinvestment return occurring in the portfolio. This principle of duration-matching together with rebalancing procedures that are used over time allow us to lock in rates and minimize the reinvestment risk that is associated with the maturity-matching strategy.

SINGLE-PERIOD IMMUNIZATION

The most straightforward approach to funding a single-period liability five years from today is to purchase a five-year, zero-coupon bond maturing on the liability payment date. Regardless of future fluctuations in interest rates, the bond, or portfolio of bonds, will be price insensitive (or immune) to changes in rates as the zero-coupon securities mature at par on the payment date. Because zero coupons have durations equal to their maturities, the five-year zero-coupon bonds both cash-match and duration-match the single-period liability payment.

If zero-coupon bonds have insufficient yield, a portfolio of *coupon-bearing* Treasury, agency, and corporate bonds can be immunized to fund the same single-period payment only if three conditions are met: (1) the duration of the portfolio

EXHIBIT 47–1

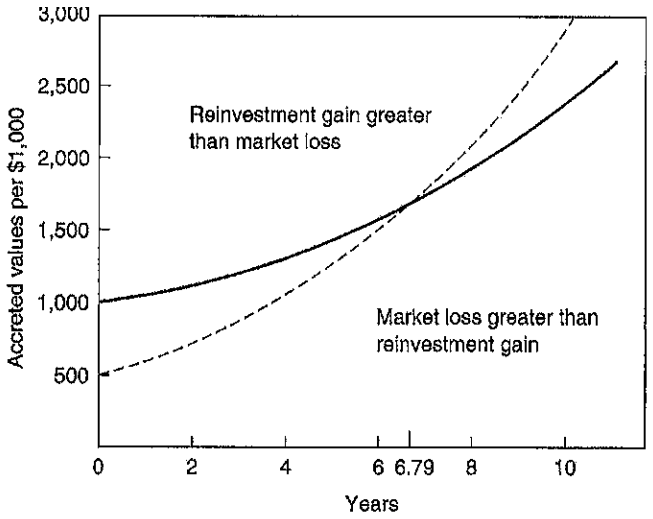
Total Return on a 9% Noncallable $1,000 Bond Due in 10 Years and Held through Various Holding Periods

Income Source	Interest Rate at Time of Reinvestment	*Holding Period in Years*					
		1	3	5	6.79[a]	9	10
Coupon income	5%	$90	$270	$450	$611	$810	$900
Capital gain or loss		287	234	175	100	39	0
Interest-on-interest		1	17	54	105	191	241
Total return (and yield)		$378 (37.0%)	$521 (15.0%)	$679 (11.0%)	$816 (9.0%)	$1,040 (8.5%)	$1,141 (8.2%)
Coupon income	7	$90	$270	$450	$611	$810	$900
Capital gain or loss		132	109	83	56	19	0
Interest-on-interest		2	25	78	149	279	355
Total return (and yield)		$224 (22.0%)	$404 (12.0%)	$611 (10.0%)	$816 (9.0%)	$1,108 (8.6%)	$1,225 (8.5%)
Coupon income	10	$90	$270	$450	$611	$810	$900
Capital gain or loss		0	0	0	0	0	0
Interest-on-interest		2	32	103	205	387	495
Total return (and yield)		$92 (9.0%)	$302 (9.0%)	$553 (9.0%)	$816 (9.0%)	$1,197 (9.0%)	$1,395 (9.0%)
Coupon income	10	$90	$270	$450	$611	$810	$900
Capital gain or loss		–112	–95	–75	–56	–18	0
Interest-on-interest		2	40	129	261	502	647
Total return (and yield)		$20 (2.0%)	$215 (6.7%)	$504 (8.5%)	$816 (9.0%)	$1,294 (9.7%)	$1,547 (9.8%)

[a]Duration of a 9% bond bought at par and due in 10 years.

EXHIBIT 47–2

"Offsetting Forces" Principle (9% Coupon, 30-Year Maturity Bond, Rates Rise Instantly from 9% to 15%, Reinvestment Rate Is 15%)



of coupon bonds must be set equal to the five-year horizon, (2) the market value of assets must be greater than the present value of liabilities, and (3) the dispersion of the assets must be slightly greater than the dispersion of the liabilities. That is,

1. $\text{Duration}_{\text{Assets}} = \text{duration}_{\text{Liabilities}}$
2. $\text{PV}_{\text{Assets}} > \text{PV}_{\text{Liabilities}}$
3. $\text{Dispersion}_{\text{Assets}} > \text{dispersion}_{\text{Liabilities}}$

Immunization requires that the average durations of assets and liabilities are set equal at all times. Unfortunately, simple matching of durations is not a sufficient condition.

Consider both a $200,000 par-value zero-coupon five-year bond in a 9% rate environment and a $1 million five-year single-period liability. Obviously, the durations of both the assets and liabilities are matched because they are both zero-coupon five-year obligations. However, a $200,000 par-value zero-coupon five-year bond (with a market value of $128,787) cannot realistically compound to $1 million in five years. The required annual rate to compound to $1 million in five years is almost 67%. In a 9% rate environment, $643,937 is required in market value of assets to compound to $1 million in five years.

Therefore, a second condition for immunization is necessary: The market value of assets must be greater than or equal to the present value of liabilities, using the internal rate of return (IRR) of the assets as the discount factor in present-valuing the liabilities. The assets, when compounded at the "locked-in"

immunized rate of 9%, will grow to equal or exceed the future-value immunized target of $1 million in this example.

To meet a target duration of 6.79, a portfolio could be constructed as either (1) a barbell of roughly equal amounts of bonds with zero and 13 duration, (2) an even ladder of equal amounts of bonds with zero through 13 duration, or (3) a bullet of only 6.79 durations. Because the duration calculation assumes a parallel shift in the yield curve, the barbell structure incorporates the greatest amount of yield-curve risk by concentrating cash flows on both ends of the curve. If the yield curve is positive or inverted, the barbell structure will violate the assumption of a flat curve more than the even ladder or bullet structure. On the other hand, the bullet structure, by concentrating cash flows at a single maturity point, incorporates a flat slope over the relevant range on the yield curve.

For single-period immunization, a bullet maturity structure with tight cash flows around the liability date generally is preferred to an even ladder or barbelled portfolio because of the reduced risk exposure to the yield curve becoming steeper or twisting. In fact, to eliminate the risk of pathologic shifts in yields, the investor could tighten the cash flows still further and purchase a zero-coupon bond to cash-flow-match the single-period liability. Short of that, a bullet structure is the least risky, and the barbell the most risky.

Therefore, for immunization, the third condition of controlling the degree of barbelling must be incorporated into the process of structuring a portfolio. The measure used to control the barbelling is dispersion—a measure of the variance of cash flows around the duration (D) of a bond. The mathematical formula for dispersion is as follows[3]:

$$\text{Dispersion} = \frac{\sum (t_1 - D)^2 \text{PV}(\text{CF}_i)}{\sum \text{PV}(\text{CF}_i)}$$

The dispersion of a zero-coupon bond therefore is zero, whereas the dispersion of the long-term coupon U.S. Treasury bond can exceed 100.

REBALANCING PROCEDURES

As time passes, the single-period immunized portfolio must be rebalanced so that the duration of the portfolio is always reset to the remaining life in the planning period to ensure the offsetting effects of capital gains with reinvestment return. This rebalancing procedure requires that the coupon income, reinvestment income, matured principal, and proceeds from possible liquidation of longer bonds be reinvested into securities that maintain the duration equal to the remaining life in the planning period. Because of the multiple rebalancings required throughout the

3. This measure, commonly referred to as M^2, was first developed in H. Gifford Fong and Oldrich Vasicek, "A Risk Minimizing Strategy for Multiple Liability Immunization," *Journal of Finance* (December 1984), pp. 1541–1546.

planning period, the bond portfolio is continually maintained in a duration-matched state and therefore should achieve its target return in spite of periodic shifts in rates.

An immunized bond portfolio therefore can be constructed once a time horizon is established. Because duration is inversely related to both the prevailing yields and the coupon rate, it may not be possible to immunize a portfolio beyond a certain number of years using only coupon-bearing securities. For example, when bond market yields reached their historic highs in 1981, it was not possible to immunize a bullet liability beyond seven years in the taxable markets with current-coupon securities. In an 8% rate environment, the maximum lock-up period would be closer to 12 years. However, the use of zero-coupon securities with long maturities and durations can allow the investor the opportunity to lengthen the planning period over which he or she can lock in rates.

The actual targeted return on an immunized portfolio will depend on the level of interest rates at the time the program is initiated. Though bond values may, for example, decline as interest rates rise, the future value of the portfolio (or security) based on the new higher reinvestment rate and lower principal value should still correspond to the original targeted yield. Duration is the key to controlling the equal offset of reinvestment income with asset value as interest rates fluctuate.

The important point to remember is this: *The standard deviation of return on an immunized portfolio will be much lower over a given horizon than that on a nonimmunized portfolio—whether measured around a sample mean or promised yield.* With interest-rate risk minimized (when held over an assumed time horizon), the performance of the immunized portfolio is virtually ensured, regardless of reinvestment rates.

MULTIPERIOD IMMUNIZATION

In the discussions so far we have explained how the three conditions are required to create a single-period immunized portfolio. These conditions can be extended to create an immunized portfolio that will satisfy the funding requirements of multiple-period liabilities, such as the monthly payouts to the retired-lives portion of a pension plan.

If a liability schedule were composed of 30 annual payments, it would be possible to create 30 single-period immunized portfolios to fund that schedule. If we then analyzed the overall duration of the 30 asset portfolios, it would equal the duration of the liabilities. As long as the dispersions of assets and liabilities are closely matched and the asset value is greater than the present value of liabilities, then the liability schedule should be fully funded and the portfolio immunized.

Calculating the duration of multiperiod liabilities is not as straightforward as calculating the duration of a single-period liability, where the remaining time in the planning horizon is the liability duration. With multiple payout periods, the liability duration is derived by using, as the discount factor, the IRR on the assets. Of course, the IRR of the assets is not determinable unless we know the precise portfolio, its duration, and its dispersion.

As a result of this simultaneity problem, the construction of an immunized portfolio is an iterative process whereby an IRR guess for the portfolio is advanced; the durations and dispersion of the liabilities are then calculated based on the IRR guess; an optimal immunized portfolio is simulated to match the duration and dispersion estimates; the portfolio IRR is then compared with the estimated IRR; and if they differ, a new IRR estimate is advanced and the procedure repeated.

In the absence of strict cash matching, it is anticipated that some liabilities will be met through a combination of asset cash flows *and* asset sales. In this regard, immunization introduces an element of market risk into the asset/liability equation that is only minimally present under a dedicated strategy.

The degree to which market risk can be limited and the cost savings of immunization thereby justified on a risk-adjusted basis depends in large part on one's ability to characterize correctly the price response of the bonds in the portfolio to changes in interest rates. This issue is especially critical when bonds containing embedded options—such as mortgages and callable corporates—are part of the asset mix and is best resolved by appealing to option-adjusted bond analytics for the relevant bond durations.

Rebalancing Procedures for Multiperiod Portfolios

Just as with a single-period immunized portfolio, a multiperiod portfolio must be rebalanced whenever one of the three conditions is violated. If, for example, the asset and liability durations were to wander apart over time, then the portfolio must be rebalanced to return it to a duration-matched state.

In a multiperiod portfolio, the durations will tend to wander whenever a liability payment comes due. An extreme example might be a $10 million bullet liability due in one month (almost zero duration) and a $10 million bullet liability due in 10 years. The average duration of the two liabilities will be about 5.

One month from now, the one-month liability will be extinguished, and the remaining liability will be 9 years and 11 months. Since the asset portfolio has a duration of roughly 5 to match what was an average duration liability of 5, the sudden shift in liability duration from 5 to approximately 10 will cause a major duration mismatch and will need to be rebalanced.

APPLICATIONS OF THE IMMUNIZATION STRATEGY

As indicated in Exhibit 47–3, the major applications of the immunization strategy have been in the pension, insurance, banking, and thrift industries.

The pension market has made widespread use of both single-period and multiperiod immunization. Single-period immunization generally is employed as an alternative to the purchase of a guaranteed investment contract (GIC) from an insurance company. Both vehicles seek to lock in today's prevailing rates over a finite planning horizon. Immunization has the advantage of liquidity, as the portfolio is composed of

EXHIBIT 47–3

Applications for Immunization

	Market		
	Pension	**Insurance**	**Banking and Thrift**
Single period	Asset strategy (GIC alternative)		
Multiperiod	Funding retired-live payouts	Funding GIC and structured settlements	GAP management Matched growth
	Single-premium buyouts	Portfolio insurance	Portfolio insurance
	Portfolio insurance		

marketable securities. GICs are privately written contracts between plan sponsor and insurance company and are not generally traded in the secondary market.

The additional benefit of an immunized portfolio is that the portfolio manager can take advantage of market opportunities in structuring and rebalancing these portfolios by including securities in the portfolio that are attractive on a relative-value basis. Investors can actively position portfolios in sectors and credits they perceive to be cheap or upgrade candidates. By actively positioning the immunized portfolio, investors can add incremental value to the portfolios and potentially outperform the illiquid GIC over a fixed planning horizon.

The pension market also has made widespread use of multiperiod immunization. Multiperiod immunization generally is employed to fund a schedule of expected benefit payouts to the retired-lives portion of a defined-benefit plan. As explained in the next chapter on cash-flow matching, by matching the duration of an immunized portfolio with corresponding liabilities, the plan sponsor can lock in prevailing rates, raise its actuarial interest-rate assumption, and reduce cash contributions to the pension fund.

The insurance market also has made widespread use of the multiperiod immunization strategy for its fixed-liability insurance products such as GICs and structured settlements. Because GIC, structured settlement, and single-premium buyout assets and liabilities generally are segmented from general account assets and liabilities, the entire line of business can be immunized to minimize the interest-rate risk and lock in a spread. Again, these portfolios can be actively positioned to take advantage of market opportunities.

Lastly, banks and thrifts have made extensive use of the multiperiod immunization strategy to assist in the management of their asset/liability gap and to ensure future duration-matched growth of assets and liabilities. *Technical Bulletin 13* (TB-13) mandated for the thrift industry that the interest sensitivity of a company's

assets be similar to the interest sensitivity of its liabilities. For thrifts whose durations are not closely matched, their capital requirements will be increased.

VARIATIONS TO IMMUNIZATION

There are several variations or enhancements to the immunization strategy, including combination-matching; contingent immunization; immunization with futures, options, mortgages, or swaps; and stochastic duration-matching.

The most popular variation of the immunization strategy is *combination-matching,* also called *horizon-matching.* A combination-matched portfolio is one that is duration-matched with the added constraint that it be cash-matched in the first few years, usually five years. The advantages of combination-matching over immunization are that liquidity needs are provided for in the initial cash-flow-matched period. Also, most of the positive slope or inversion of a yield curve tends to take place in the first few years. By cash-flow matching the initial portion, we have reduced the risk associated with nonparallel shifts of a sloped yield curve.

The disadvantages of combination-matching over immunization are that the cost is slightly greater and the swapping discretion is constrained. The freedom to swap a combination-matched portfolio is partially hampered not only because the asset durations must be replaced in a swap but also because the cash flows in the initial five-year period must be replaced as well.

A variant strategy to immunization is *contingent immunization.* The contingent immunization strategy is a blend of active management with immunization such that a portfolio is actively managed with a lower floor return ensured over the horizon.[4]

The floor return, or safety net, is a rate set below the immunized rate, allowing managers discretion to actively position their portfolios. If managers incorrectly position their portfolios and the market moves against them, the portfolios can still be actively managed. If the market continues to move against the portfolios and the floor return is violated, then managers must commit to immunized portfolios to ensure the floor return over the remainder of the horizon.

Contingent immunization requires an abrupt change in management strategy at the moment the floor return is violated. With dynamic asset allocation (portfolio insurance), the change in strategy is gradual. In this instance, managers gradually shift out of risky assets into riskless assets to avoid violating minimum return requirements. An actively managed bond portfolio or equity portfolio is the risky asset. An immunized portfolio, with duration matched to the holding period, can serve as the riskless asset. Overall, the performance of the portfolio of risky and riskless assets replicates the performance that would be obtained were a put option added to the risky portfolio. This synthetic put

4. See Martin L. Leibowitz and Alfred Weinberger, "The Uses of Contingent Immunization," *Journal of Portfolio Management* (Fall 1981), pp. 51–55.

gives the portfolio maximum upside potential consistent with a prespecified level of protection on the downside.

Immunized portfolios also can be created with the use of futures contracts to replicate the interest sensitivity of an immunized duration. In this form, a desired portfolio can be selected without regard to a target duration, and futures contracts can then be used to replicate the price sensitivity of an immunized portfolio at the desired duration.

Options also can be used with immunized portfolios to enhance returns over a specified horizon. Through the use of covered call writing or long put or call positions, managers can enhance returns over a specified horizon.

Finally, CMO PAC bonds sometimes are used in immunized portfolios to enhance returns. Though they are mortgage derivatives, their cash flows are certain across a wide band of interest-rate scenarios (prepayment speeds). As such, they can enhance performance as long as their use is actively monitored.

CONCLUSION

Bond immunization is an important risk-control strategy used by the pension fund, insurance, banking, and thrift industries. In today's volatile markets, it is imperative that all asset/liability gaps be intentional. Immunization provides the tools to measure the interest-rate risk position an institution or a fund is taking with respect to its liabilities; it also provides the tools to minimize that risk when a minimum gap is desired.