Volume 2 – IEC Counsel Book of Documents

Date: March 14, 2014

NFAT Review
Subject: Load Forecast and DSM

INDEX

TAB 1

life of the assets in question and because of the considerable uncertainty over that life. Shortening the time horizon is akin to applying a 100% discount rate or equivalently, assigning a zero value to long run impacts...even very large ones...with certainty. This is clearly in conflict with the idea that there is severe uncertainty in the long run. Instead, it is best to recognize and incorporate uncertainty over this time period in the evaluation. It is also relevant to note that, with discounting, impacts in the far future have a diminishing impact. For example, at a 5.05% discount rate, impacts in 50 years are discounted 93% and impacts in 78 years are discounted 98%.

2.2.9 Treatment of Optionality

On Page 3-6 of his report, Mr. Bowman states: *"Hydro's analysis in the NFAT fails to fully reflect the impacts of optionality and adaptation....Hydro attempts to address the concept of optionality and adaptation in PUB/MH-I-279, but this cursory analysis fails to give the concept the profile required as a key planning tool."*

Mr. Bowman points out the potential significance of optionality and adaptation, particularly in this long-term resource planning context. However, his characterization of MH's work as "cursory" especially given the complexities in conducting and communicating this form of analysis is inaccurate. In practice, economic evaluation and uncertainty analysis rarely begin with or focus on optionality. In many cases, optionality is treated only conceptually and qualitatively.

In contrast, MH put considerable emphasis in the filing on flexible pathways in addition to fixed plans, and on recognition that events going forward should be monitored over time, updated analysis conducted and adjustments made. Chapter 14 of the NFAT filing discusses the importance of optionality, and implications for the choice of resource plan. The attachment to PUB/MH I-279 provides additional analytical details.

3. Inputs

3.1 *Overview*

The second step in uncertainty analysis is collecting and processing the probabilistic data necessary for the analysis. As described in the Formulation section, the formal uncertainty analysis focused on three major uncertainties:

- Energy Prices
- Capital Costs
- Discount Rate (and other Economic Indicators)

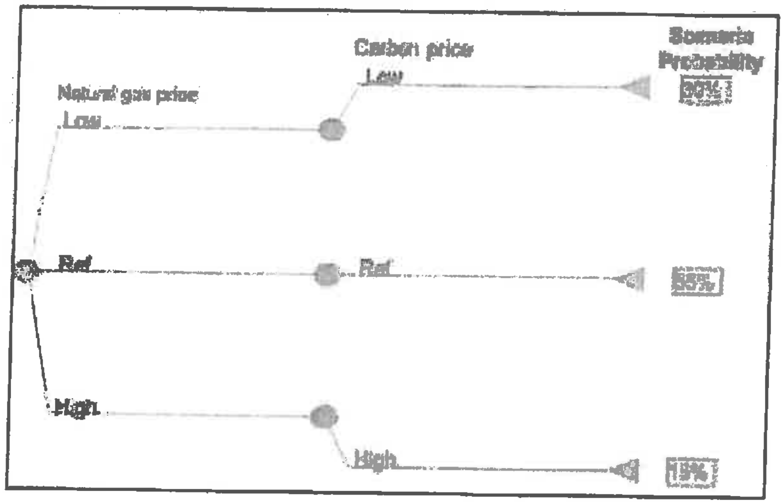
Under Dr. Borison's direction, Navigant assisted MH in modeling all three uncertainties, with a particular focus on energy prices and discount rate.

Based on both historical data and expert judgment, the three uncertainties were modeled as independent (or uncorrelated). Following sound analytical principles and common practice, each uncertainty was modeled with three discrete scenarios over the forecast time horizon, each scenario with an associated probability. This is described further in Appendix 9.3 of the NFAT filing.

3.1.1 Energy Prices

Appendix 9.3 of the NFAT filing describes the approach taken to modeling uncertainty in energy prices. With assistance from Dr. Borison, probabilistic natural gas, electricity and carbon price forecasts were developed using historical data, model results and expert judgment from leading sources. Natural gas prices and carbon prices were assumed to be independently uncertain. Electricity prices were assumed to be deterministically dependent on natural gas and carbon prices. These relationships are consistent with analytical principles and empirical evidence. With three scenarios each for natural gas prices and carbon prices, there are nine scenarios of energy prices.

For computational efficiency, these nine scenarios were summarized with three based on matching the impact of these uncertainties on levelized electricity prices. The three scenarios and their probabilities are displayed below. The mean and variance of levelized electricity prices in the nine original scenarios and three modified scenarios are very similar, indicating that the three scenarios and associated probabilities do a good job of representing the underlying uncertainty in energy prices.



For more details on the approach for developing probabilistic inputs, see PUB/MH I-161.

3.1.2 Capital Costs

Appendix 9.3 describes the approach taken to modeling uncertainty in capital costs. Using standard contingency analysis, MH developed low, reference and high forecasts for capital costs for generation and transmission technologies. Based on common industry practice, these costs were assumed to be deterministically linked. Probabilities were assigned using expert judgment from Manitoba Hydro specialists.

3.1.3 Discount Rate

Appendix 9.3 describes the approach taken to modeling uncertainty in discount rates. In many applications, discount rates reflect the known cost of capital. In this long-term application, MH developed low, reference and high forecasts reflecting the potential sustained cost of capital over the time horizon. With assistance from Dr. Borison, probabilities were assigned based on analysis of historical data, model results and expert judgment from leading sources.

3.2 *Concerns*

Various concerns have been raised with respect to the uncertainty analysis inputs used by Manitoba Hydro. This section outlines these concerns and our responses.

3.2.1 Use of Three Point Probability Distributions

On Page 1 of his report, Mr. Simpson states: "*...the 'probability distributions' describing the range of possible outcomes... are limited to three points, representing low, reference (expected) and high outcomes or scenarios. The*

NFAT refers the reader to Appendix 9.3 for details of the formation of these scenarios and their associated probabilities, but the details are sketchy."

The use of three-point probability distributions to describe uncertainty in individual input factors to economic/financial evaluation is well-established in both theory and practice. Miller & Rice and Jim Smith discuss the accuracy of various discrete approximations, including three-point probability distributions.[9] [10] (Note that with three uncertainties, each with three outcomes, there are a total of 3x3x3 or twenty-seven distinct scenarios, each with its own probability.) In general, a well-designed three-point probability distribution does a good job of representing an underlying continuous distribution in most investment-related applications. MH was careful to calibrate the probability distributions to historical data, model results and expert judgment. Further information on this process is provided in PUB/MH I-161.

3.2.2 Use of Dated Probability Assessments

On Page LCA-11 of their report, LCA states: *"the assessment of probabilities that MH used to develop its probabilities for the uncertainty scenarios begins with opinions of experts at a given point in time as the key inputs. The probabilities are very important input assumptions that can be changed to reflect updated information or differences of opinions. LCA is concerned that the perspective of the experts that was captured in the probabilistic analysis has likely changed in the two years since the analysis was conducted."*

With assistance from Navigant, MH used a mix of expert judgment from leading sources, historical data and model forecasts to generate inputs for the uncertainty analysis. It is true that all these elements can change to a greater or lesser degree over time. In general, MH used several sources of information rather than one, and used the latest publically-available information at the time. Furthermore, sensitivity analysis was conducted on key variables. This should help reduce the impact of changes in expert opinion. Chapter 12 of the NFAT filing describes the results of updating several of the key inputs.

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[9] Allen C. Miller and Thomas R. Rice, *Discrete Approximations of Probability Distributions*, Management Science, Volume 29, Number 3, pp. 352-362, 1983.

[10] James E. Smith, *Moment Methods for Decision Analysis*, Management Science, Volume 39, Number 3, pp. 340-358.

4. Outputs

4.1 *Overview*

The third step in uncertainty analysis is generating and interpreting outputs. These outputs are designed to show how each alternative performs in a world of uncertainty. In this context, there are four key outputs:

- Scenario Value
- Expected Value
- Risk
- Value of Optionality

Under Dr. Borison's direction, Navigant advised MH in developing, displaying and understanding these four outputs, with a particular focus on risk and optionality.

4.1.1 Expected Value

In uncertainty analysis, the most basic output of interest is the metric or value in each scenario – in this case, NPV. These outputs are displayed in color-coded quilts. These values reflect what is known as the utilitarian approach. In this approach, the value for each alternative in each scenario is in reference to the same fixed base - a single specified alternative and scenario, such as "All Gas – Ref- Ref- Ref." Each number reflects the incremental value created over this fixed base. This is in contrast to the regret approach where value for each alternative in each scenario is in reference to a variable base – a specified alternative such as "All Gas" in the same scenario. The utilitarian and regret approaches are compared in more detail in MIPUG/MH II-004a and MIPUG/MH II-009a.

4.1.2 Expected Value

In uncertainty analysis, the single most important output is typically the expected value or mean. As explained in Appendix 9.3 of the NFAT filing, the expected value in this analysis is the sum of each scenario NPV x the probability of its occurrence. These outputs are shown both in tabular and graphical format.

4.1.3 Risk

In addition to the expected value, uncertainty analysis produces a variety of outputs showing the risk associated with each alternative. There are a variety of tabular and graphic devices for displaying these outputs including scatter plots, box and whisker plots and S-curves. Appendix 9.3 and Chapter 10 of the NFAT filing display many such outputs. In non-financial applications, these outputs are generally more useful than standard summary risk measures such as 10-90 ranges and standard deviations.

In general, S-curves are considered the most useful of these outputs because they display the most information and can be used to answer a broad range of risk questions. S-curves and similar outputs are described in Appendix 9.3 and Chapter 10 of the NFAT filing. Important properties of S-curves are described in CAC/MH II-057 and CAC/MH II-058.

4.1.4 Value of Optionality

In uncertainty analysis, the added value of learning and adaptation, optionality, can be important. This value compares the expected value and risk of the best fixed plan and the best flexible pathway. The difference is the value of optionality. This value is discussed in Chapter 14 of the NFAT filing and quantified in the attachment to PUB/MH I-279.

4.2 *Concerns*

Various concerns have been raised with respect to the uncertainty analysis outputs used by Manitoba Hydro. This section outlines these concerns and our responses.

4.2.1 Use of Utilitarian Rather Than Regret Approach

LCA makes the following statements in their report about the approach taken to comparing the impact of different alternatives in different scenarios:

- Page LCA-11: "*MH's method of comparing cases and presenting the results is unconventional and can be misleading, as a result. MH's method compares all cases to a single case, All Gas using reference case assumptions on discount rate, energy prices, and capital costs. This does not compare the performance of All Gas to the other plans with comparable sets of assumptions. We view such comparison to be more informative and a more conventional practice for this method of analysis. LCA has developed a model that does the comparative analysis of MH's cases. The LCA Methodology provides a comparative analysis across plans on consistent assumptions of uncertain parameters. Using this method with MH's economic analysis results and probability assessments, the comparative assessments of the plans are quite different than depicted in the MH S-Curve analysis.*"
- Page 9A-61: "*LCA, when utilizing similar analysis techniques of probabilities and S-curve representations, compares case results of each plan to the base case (All Gas) under the same scenario. The NPV differences that result can be interpreted in this manner, a negative number means the All Gas Plan under that scenario has lower costs than the specific plan being examined under the same set of assumptions of the uncertain variables. When the comparative NPV metric is positive, the alternative plan is lower cost on this NPV basis at the end of 78 years. By comparing a given plan's result to a base case, the quilt and S-curves are providing a view of "the risk of regret." In other words, would the decision maker look back and regret the choice made if events prove to be different than the reference case assumptions and the investment does not provide the value expected? However, when doing this type of analysis, it is important to ensure that the comparisons being made are, in fact, useful to the decision maker in question. We believe that MH's methodology of comparing all 27 scenarios of a Development*

Plan's potential outcomes to a single (reference) point in the base case does not indicate the most important element, which plan is economic for a given scenario."

As LCA indicates, there are a variety of approaches that can be taken in comparing the impacts of different alternatives scenarios, and using this comparison for decision-making. In general, these approaches are either descriptive or prescriptive. Descriptive approaches provide formal rules for how individuals (and organizations) actually make decisions; prescriptive approaches provide formal rules for how individuals (and organizations) should make decisions.

The dominant prescriptive approach is called expected utility or utilitarian. It has broad and deep analytical foundations, and is a widely accepted and well-regarded approach for improved decision-making. In a review of various approaches, Dr. L. Robin Keller said simply: *"Expected utility is well suited as a prescriptive model of decision-making under risk."*[11] This is the approach taken by MH with Navigant's assistance.

In the utilitarian approach, each outcome represents the difference between a specific plan in a specific scenario and a fixed base value. MH considered and rejected the alternative "regret" approach (except as a supplement) where plan outcomes are compared within a specific scenario on a scenario-by-scenario basis. (Contrary to what LCA indicates, the approach used by MH is not termed "regret.") Interestingly, LCA's own report reveals the problems with their suggested approach since, for example, it shows the All Gas alternative as apparently without risk. (See Figure 9-28 in the LCA report.) This is despite the significant uncertainty regarding natural gas prices and other factors. Other shortcomings of the regret approach are described in MIPUG/MH II-004a and MIPUG/MH-1-009a. Tellingly, advocates of the regret approach as a descriptive tool do not typically recommend it as a guide for rational decision-making. For example, Dr. Bell, a leading author on the regret approach, says: *"[This]...paper does not suggest that people ought to make financial tradeoffs to avoid disappointment."*[12]

4.2.2 Interpretation of Impact Magnitude and Timing

On Page 9A-21 of their report, LCA states: *"The economic differentials at the end of 78 years between the alternative development plans is relatively small compared to the scale of the investments proposed in the Preferred Development Plan."*

On Page 9A-80 of their report, LCA states: *"the expected benefits from the PDP, relative to the alternative plans, are largely derived from upside opportunities after the first 50 years"*.

As noted in Section 2.2.8, the 78-year NPV metric is a good measure of the incremental economic value created by each alternative (importantly, above and beyond any investment cost). Consequently,

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[11] L. Robin Keller, *The Role of Generalized Utility Theories in Descriptive, Prescriptive and Normative Decision Analyses*, Information and Decision Technologies, Vol. 15, pp. 259-271, 1989.

[12] David E. Bell, *Disappointment in Decision Making Under Uncertainty*, Operations Research, Jan/Feb 1985.

TAB 2

SUBJECT: Economic Analysis

REFERENCE: Technical Appendix 9A, Page 9-64, Figure 9-26

QUESTION:

Please confirm that the values for each scenario provided in Figure 9-26 are the same as those provided in Appendix 9.3, Figure 2.7.2 of the NFAT submission. If not confirmed, please reconcile the two figures.

RESPONSE:

LCA has reviewed the table provided in Appendix 9.3, Figure 2.7.2 of the NFAT submission. LCA can confirm that Figure 2.7.2 has the same values as Table 2 of Manitoba Hydro's Executive Summary of the NFAT submission. Both of these tables display comparisons of each plan compared with the All Gas Plan distinctly for each scenario, calculated in the same manner as Figure 9-26 of LCA Technical Appendix 9A. The calculation is identical to what is referred to as the LCA Methodology through Appendices 9A and 9B. Thus LCA can confirm that the values for each scenario in Figure 2.7.2 in Appendix 9.3 and Table 2 in the Executive Summary of the NFAT submission both match the values of Figure 9-26 in Technical Appendix 9A.

TAB 3

La Capra Associates

PUBLIC VERSION

January 24, 2014, Page 9A-64

The Commercially Sensitive Information contained within this report has been redacted in accordance with the protective order.

[illegible]	[illegible]	Capital Costs	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	All Gas	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Low (20%)	Low (15%)	High (30%)	1202	401	1188	489	625	584	853	537	0	[illegible]	281	-12	734	514	-500
		Ref (50%)	1439	872	1433	911	919	924	1172	883	0	[illegible]	517	203	640	567	-100
		Low (20%)	1355	1217	1544	1224	1072	1278	1356	1148	0	-1419	657	474	592	629	286
	Ref (50%)	High (30%)	-1492	[illegible]	-267	-1561	-729	-2047	-448	-1554	0	[illegible]	-749	[illegible]	-834	-1068	-1732
		Ref (50%)	-1137	-1768	49	-1362	-393	-1576	-114	-1254	0	-1686	-487	-1704	-790	-913	-1383
		Low (20%)	-730	-1335	224	-993	-202	-1223	87	-926	0	-1072	-342	-1428	-744	-779	-1085
	High (35%)	High (30%)	[illegible]	[illegible]	-1001	[illegible]	-1386	[illegible]	-1087	[illegible]	0	-2008	-1179	[illegible]	-1498	-1721	[illegible]
		Ref (50%)	[illegible]	[illegible]	-648	[illegible]	-1029	[illegible]	-744	[illegible]	0	-1324	-923	[illegible]	-1305	-1521	-1894
		Low (20%)	-1938	[illegible]	-440	-1914	-817	[illegible]	-534	-1787	0	-869	-770	-2163	-1292	-1352	-1938
Ref (60%)	Low (15%)	High (30%)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	-2154	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Ref (50%)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	-782	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Low (20%)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	153	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Ref (60%)	High (30%)	1141	861	700	861	736	823	1012	775	0	[illegible]	823	434	684	828	148
		Ref (50%)	1695	1427	1097	1360	1091	1295	1346	1215	0	-775	807	806	730	704	531
		Low (20%)	[illegible]	1860	1272	1729	1282	1648	1547	1547	0	-262	1031	1313	784	917	834
	High (30%)	High (30%)	-1040	-1306	-268	-942	-310	-1150	-50	-917	0	-1390	-218	-1100	-429	-546	-538
		Ref (50%)	-432	-669	86	-416	47	-668	292	-441	0	-714	39	-739	-325	-384	-558
		Low (20%)	16	-196	258	-30	259	-249	503	-87	0	-260	184	-441	-253	-175	-262
High (15%)	Low (15%)	High (30%)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	-284	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Ref (50%)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	1087	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Low (20%)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	2073	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Ref (60%)	High (30%)	[illegible]	[illegible]	1871	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	-609	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Ref (50%)	[illegible]	[illegible]	2187	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	288	2099	[illegible]	[illegible]	[illegible]	[illegible]
		Low (20%)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	912	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	High (30%)	High (30%)	994	1027	482	1019	783	961	1036	317	0	-477	807	725	747	743	528
		Ref (50%)	1602	1664	835	1546	1141	1492	1379	1352	0	-1	1064	1126	850	942	909
		Low (20%)	2050	2136	1043	1531	1353	[illegible]	1849	[illegible]	0	453	1280	[illegible]	943	1111	1205

Figure 9-26: Probabilistic Analysis Quilt 78 Year with LCA Methodology for Determining Comparisons to All Gas Plan

TAB 4

1 **Table 2** NET PRESENT VALUE BENEFITS OF ALTERNATIVE DEVELOPMENT PLANS UNDER 2012 HIGH, REFERENCE AND LOW

2 ASSUMPTIONS FOR ENERGY PRICES, CAPITAL COSTS AND DISCOUNT RATES

Pathway			Pathway 1			Pathway 2		Pathway 3			Pathway 4			Pathway 5	
Development Plan			1	7	8	2	10	4	13	11	6	15	12	5	14
			All Gas	SCGT/C26	CCGT/C26	K22/Gas	K22/C29	K19/Gas24 /250MW	K19/C25 /250MW	K19/C31 /250MW	K19/Gas31 /750MW	K19/C25 /750MW	K19/C31 /750MW	K19/Gas25 /750MW	K19/C25 /750MW
														WPS Sale & Investment	
Energy Prices	Discount Rates	Capital Costs	Millions of 2014 NPV dollars												
Low	Low	H	0	734	514	251	-22	853	584	537	625	401	489	1108	1202
		Ref	0	648	567	517	263	1172	925	883	919	872	911	1453	1639
		L	0	592	620	657	474	1357	1178	1149	1072	1217	1224	1544	1955
	Ref	H	0	-834	-1068	-749	-2076	-448	-2047	-1658	-729	-2353	-1861	-267	-1692
		Ref	0	-790	-913	-487	-1704	-114	-1576	-1258	-393	-1768	-1362	49	-1137
		L	0	-744	-779	-342	-1428	87	-1223	-926	-202	-1335	-993	224	-730
	High	H	0	-1488	-1721	-1179	-2863	-1087	-3233	-2617	-1386	-3587	-2826	-1001	-3014
		Ref	0	-1385	-1521	-923	-2462	-744	-2701	-2141	-1029	-2950	-2299	-648	-2406
		L	0	-1292	-1352	-778	-2165	-534	-2303	-1787	-817	-2478	-1914	-440	-1958
Ref	Low	H	0	3153	3310	2502	3174	3218	5220	4660	2073	5511	5013	2910	5868
		Ref	0	3167	3359	2768	3458	3557	5551	5026	3157	5983	5435	3155	6304
		L	0	3111	3417	2909	3669	3771	5834	5291	3400	6377	5749	3266	6620
	Ref	H	0	694	628	625	434	1012	823	775	756	841	861	780	1141
		Ref	0	738	784	887	806	1346	1295	1215	1091	1427	1360	1097	1696
		L	0	784	917	1031	1083	1547	1648	1547	1282	1860	1729	1272	2103
	High	H	0	-429	-544	-218	-1140	-50	-1199	-917	-310	-1306	-942	-268	-1040
		Ref	0	-325	-344	39	-739	292	-668	-441	47	-669	-416	86	-432
		L	0	-233	-175	184	-441	503	-269	-87	259	-196	-30	294	16
High	Low	H	0	6079	6411	4936	2790	5742	10254	9230	5428	10890	9844	4740	10807
		Ref	0	5993	6405	5202	3074	6062	10595	9570	5722	11361	10266	4905	11244
		L	0	5947	6318	5342	3185	6246	10843	9841	5975	11706	10590	5046	11550
	Ref	H	0	2398	2494	2099	3172	2556	3903	3458	2285	4151	3729	1871	4098
		Ref	0	2442	2649	2361	3543	2850	4375	3905	2620	4736	4228	2187	4553
		L	0	2489	2703	2505	3920	3091	4728	4230	2812	5120	4592	2362	5050
	High	H	0	747	742	807	725	1036	961	917	783	1027	1019	482	994
		Ref	0	850	942	1064	1126	1379	1492	1392	1141	1664	1546	835	1602
		L	0	943	1111	1208	1424	1589	1891	1747	1353	2136	1931	1043	2050

Colour Legend: <-3000 | 0 | >3000

3

TAB 5

Figure 2.7.2: Red-Green Quilt; Relative to All Gas Development Plan

Energy Prices	Discount Rates	Capital Costs	All Gas	Wind/Gas	K22/Gas	K19/Gas24 /250MW	K19/[illegible]/750MW (WPS Sale & Inv)	K19/Gas31 /750MW	SCGT/C26	CCGT/26	Wind/C26	K22/C29	K19/C31/ 250MW	K19/C31/ 750MW	K19/C25/ 250MW	K19/C28/750MW (WPS Sale & Inv)	K19/C25/ 750MW
Low (30%)	Low (15%)	High (30%)	0	-3726	251	853	1188	625	734	514	-500	-22	537	489	564	1202	401
		Ref (50%)	0	-2354	517	1172	1433	919	648	567	-100	263	883	911	925	[illegible]	872
		Low (20%)	0	-1419	657	1357	[illegible]	1072	592	620	206	474	1148	1224	1178	1639	1217
	Ref (50%)	High (30%)	0	-2593	-749	-448	-267	-729	-834	-1068	-1780	-2076	-1698	-1861	-2047	[illegible]	-2353
		Ref (50%)	0	-1686	-487	-114	49	-393	-790	-913	-1389	-1704	-1258	-1362	-1576	-1692	-1768
		Low (20%)	0	-1073	-342	87	224	-202	-744	-779	-1085	-1428	-926	-993	-1223	-1137	-1335
	High (35%)	High (30%)	0	-2000	-1179	-1087	-1001	-1386	-1488	-1721	-2274	-2863	-2617	-2826	-3233	-730	-3587
		Ref (50%)	0	-1324	-923	-744	-646	-1029	-1385	-1521	-1894	-2462	-2141	-2299	-2701	-3014	-2950
		Low (20%)	0	-869	-778	-534	-440	-817	-1292	-1352	-1598	-2165	-1787	-1914	-2303	-2406	-2478
Ref (55%)	Low (15%)	High (30%)	0	-3154	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-1958	[illegible]
		Ref (50%)	0	-782	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Low (20%)	0	153	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Ref (50%)	High (30%)	0	-1683	625	1012	780	756	694	628	140	434	775	861	823	[illegible]	[illegible]
		Ref (50%)	0	-775	887	1346	1097	1091	738	784	531	806	1215	1360	1295	1141	841
		Low (20%)	0	-162	1031	1547	1272	1282	784	917	834	1083	1547	1729	1648	1696	1427
	High (35%)	High (30%)	0	-1390	-218	-50	-268	-310	-429	-544	-938	-1140	-917	-942	-1199	2103	1860
		Ref (50%)	0	-714	39	292	86	47	-325	-344	-558	-739	-441	-416	-668	-1040	-1306
		Low (20%)	0	-260	184	503	294	259	-233	-175	-262	-441	-87	-30	-269	-432	-669
High (15%)	Low (15%)	High (30%)	0	-284	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16	-196
		Ref (50%)	0	1087	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Low (20%)	0	2023	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Ref (50%)	High (30%)	0	-609	2099	[illegible]	1871	2285	2398	2434	2261	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Ref (50%)	0	298	2361	[illegible]	2187	2620	2442	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Low (20%)	0	912	2505	[illegible]	2362	[illegible]	2409	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	High (35%)	High (30%)	0	-677	807	1036	482	783	747	742	529	725	917	1019	961	[illegible]	[illegible]
		Ref (50%)	0	-1	1064	1379	835	1141	850	942	909	1126	1392	1546	1492	994	1027
		Low (20%)	0	454	1208	1589	1043	1353	943	1111	1205	1424	1747	[illegible]	1891	1602	1664
																2050	2176

TAB 6

Table 3 further analyses these results by providing the probabilistic average of the differences for each plan relative to the All Gas Plan and provides an indication of the upper and lower range of benefits each plan would be able to achieve.

Table 3 DEVELOPMENT PLAN ECONOMIC EVALUATION SUMMARY

Pathway	Pathway 1			Pathway 2		Pathway 3			Pathway 4			Pathway 5	
	All Gas with no new interconnection			Keeyask with no new interconnection		Keeyask with 250 MW new interconnection (MP Sale)			Keeyask with 750 MW new interconnection (MP Sale)			Keeyask with 750 MW new interconnection (WPS & MP Sales)	
Development Plan	1	7	8	2	10	4	13	11	6	15	12	5	14
	All Gas	SCGT/C26	CCGT/C35	K22/Gas	K22/C29	K19/Gas24 /250MW	K19/C25 /250MW	K19/C31 /250MW	K19/Gas31 /750MW	K19/C25 /750MW	K19/C31 /750MW	K19/Gas25 /750MW	K19/C25 /750MW
												WPS Sale & Investment	
	Millions of 2014 NPV dollars												
Ref-Ref-Ref NPV	0	738	784	887	806	1346	1295	1215	1091	1427	1360	1097	1696
Expected Value Difference From All Gas	0	525	529	634	418	1041	782	806	776	830	891	842	1155
90th Percentile - "Reward"	1905	1956	2070	2007	2601	2479	3180	2953	2215	3360	3220	2256	3377
10th Percentile - "Risk"	-3502	-1217	-1424	-1249	-1692	-898	-1988	-1362	-1181	-2186	-1594	-828	-1429

"Ref-Ref-Ref NPV" = results for reference scenario assumptions for energy prices, capital costs and discount rate (relative to All-Gas reference case result)

"Expected Value" = probabilistic weighted average of results for each of the 27 scenarios (relative to All-Gas Expected Value)

"90th Percentile-Reward"= 90th percentile probability upside benefit potential of that plan (relative to All-Gas reference scenario result)

"10th Percentile-Risk"= 10th percentile probability downside risk of that plan (relative to All-Gas reference scenario result)

TAB 7

REFERENCE: Chapter 10: Economic Uncertainty Analysis - Probabilistic Analysis and Sensitivities; Section: 10.1.4; Page No.: Figure 10.7

QUESTION:

Please provide a version of Figure 10.7 which is based off the quilt values in the form of Table 2 of the Executive Summary (i.e., as an incremental to the All Gas plan using equivalent input assumptions).

RESPONSE:

The alternative presentation of box and whisker plots (and s-curves) requested is not considered to be appropriate as explained below.

In its NFAT submission, Manitoba Hydro has shown two ways to measure the impact of uncertainty for each development plan, one for ease of understanding and one on which the probabilistic analysis is based.

The method known as the utilitarian approach, is to evaluate each alternative based on the direct impact that it has in each scenario; that is, without any reference to the impact of the other alternatives in that scenario. "If we choose alternative B and scenario 1 happens, our wealth will go up by $100,000." This method does not presume that we know which future scenario will occur. Manitoba Hydro's probabilistic analysis is based on this method.

The other method, which may be termed the regret approach, is to evaluate each alternative based on the relative impact that it has compared to the impact if some other alternative had been chosen instead. The other alternative is sometimes the "business as usual" or "default" choice. "If we choose alternative B and scenario 1 happens, our wealth will be $100,000 more than it would have been if we had chosen alternative A instead."

The regret approach has intuitive appeal and provides interested parties with some useful information at the high level. Consequently, it is used in Table 2 of the Executive Summary. However, it is usually viewed as being more descriptive than prescriptive. There is little support, analytical or empirical, for using the regret approach to make complex, future altering decisions.

One shortcoming of the regret approach is that it can give a distorted and misleading picture of the risk of each alternative, and consequently lead to the incorrect choice among alternatives. A simple example will help illustrate this. Consider choosing among three investment alternatives: A, B and C in a world of three scenarios: low, reference and high.

The table below shows this investment choice from the utilitarian perspective; that is, evaluating the direct impact of each alternative in each scenario.

UTILITARIAN

	Lo	Ref	Hi	EV	SD
A	100	0	-100	*0*	*100*
B	50	50	50	*50*	*0*
C	125	25	-75	*25*	*100*

Alternative A, business as usual, increases wealth by $100,000 in the low scenario, causes no change in the reference scenario, and decreases wealth by $100,000 in the high scenario. Alternative B increases wealth by $50,000 in all scenarios. Alternative C increases wealth by $125,000 in the low .scenario, increases wealth by $25,000 in the reference scenario, and decreases wealth by $75,000 in the high scenario.

The table also shows the expected value and standard deviation of each alternative. Among these three choices, it seems pretty clear that Alternative B is best. It is effectively a sure

$50,000 wealth increase. Alternatives A and C are risky, both having the possibility of sizeable losses - $100,000 and $75,000 respectively. And they have expected values of $0 and $25,000 respectively, both less than Alternative B. It's hard to see why one would not choose Alternative B.

The table below shows the same choice from the regret perspective; that is, evaluating the relative impact of each alternative in each scenario compared to the impact of Alternative A in that scenario as if we know that is the future scenario that will occur.

REGRET

	Lo	Ref	Hi	EV	SD
A	0	0	0	*0*	*0*
B	-50	50	150	*50*	*100*
C	25	25	25	*25*	*0*

By definition, Alternative A has $0 regret in each scenario. Alternative B has a loss (or regret) of $50,000 in the low scenario, a gain (or relief) of $50,000 in the reference scenario, and a gain (or relief) of $150,000 in the high scenario. Alternative C has a gain (or relief) of $25,000 in all scenarios.

The table also shows the expected value and standard deviation of each alternative. The expected values are the same as with the utilitarian approach: $0, $50,000 and $25,000 respectively. However, the standard deviations are entirely different. Based on regret, Alternative A and Alternative C appear to have no risk, and Alternative B appears to be risky – with a (regret) loss of $50,000. Using the regret approach, Alternative C looks very attractive. Somewhat lower expected value than Alternative B, but a lot less risk.

The different result in the two approaches stems from different meanings of risk. In the utilitarian case, we are referring to risk in actual wealth – having more or less money. In the regret case, we are referring to risk in the level of regret. If we choose Alternatives A and C, there is no risk in the level of regret but there is a great deal of risk in actual wealth. If we choose Alternative B, there is considerable risk in the level of regret despite the fact that there is no risk in actual wealth. This alternative is the proverbial sure thing - a risk free $50,000 addition to our wealth.

For Manitoba Hydro, we believe that the definition of risk should be based on the utilitarian approach; that is, we should evaluate alternatives based on the range of their direct economic and non-economic impacts on our stakeholders. Not based on the range in the level of regret associated with those impacts. Consequently, we do not think it useful to the process to develop graphic risk displays, such as box-and-whisker plots or S-curves, using the regret approach. We are concerned that these displays could be misleading and could inappropriately influence recommendations.

TAB 8

SUBJECT: Economic Uncertainty analysis

REFERENCE: Technical Appendix 9A; Page 9A-56, Page 9A-61, Page 9A-153; LCA Initial Expert Analysis Report Page ii, Page 11, Page 26

PREAMBLE: "In the development of the PAQ shown in Table 10.4, MH individually calculated the NPV of the net costs of each plan under each of the 27 scenarios of the uncertainty branches. ... MH, consistent with the economic analysis under the reference scenario discussed in Section II of this report, chose to use Plan 1, All Gas, as the base case. In the quilt above, the Reference-Reference-Reference All Gas case cell is 0, indicating that all values in the quilt are relative to that value. LCA does not believe this is the appropriate comparison to make in utilizing the array of results of these 12 plans across 27 scenarios for future conditions."

QUESTION:

LCA states that Manitoba Hydro's approach "is unconventional and can be misleading." Please provide any support for this view, specifically, any reputable sources for the view that the regret approach is a better way of making important resource planning decisions and any examples from reputable sources of the application of the regret approach to important resource planning decisions.

RESPONSE:

LCA has not made the assertion postulated in the question. The citations to LCA's reports included in the REFERENCE and PREAMBLE are a critique of MH's implementation of its regrets analysis. Specifically, MH has presented its version of a regrets analysis in the NFAT Submission (for example, refer to Chapter 14 – Conclusions, pages 9 to 10). The referenced citations from LCA's reports take issue with MH's method of comparing alternatives in its regrets analysis.

In LCA's Technical Appendix 9A, we provide the alternative approach to presenting the regrets analysis sought by MIPUG in IR MIPUG/MH I-009 (parts a and b). In MH's response to that request, it states:

> *"The regret approach has intuitive appeal and provides interested parties with some useful information at the high level. Consequently, it is used in Table 2 of the Executive Summary. However, it is usually viewed as being more descriptive than prescriptive. There is little support, analytical or empirical, for using the regret approach to make complex, future altering decisions.*
>
> *One shortcoming of the regret approach is that it can give a distorted and misleading picture of the risk of each alternative, and consequently lead to the incorrect choice among alternatives." (MIPUG/MH I-009a at page 2).*

MH has also offered a simple numerical example of how it believes the results of a regrets analysis could be misconstrued by a decision maker in its response to MIPUG/MH II-004a.

LCA observes that MH labels its analysis as "the regret approach" in its response to MIPUG/MH I-009a and it has featured its regrets analysis in its Executive Summary and Conclusions in its NFAT Submission. It then offers these critiques and characterizations of the limitations of the regret approach. These positions are contradictory.

Further, we note that in two instances, MH included the same analysis method used by LCA. Table 2 of the Executive Summary and Table 2.7.2 in Appendix 9.3 include results of the analysis in this manner. In all other regrets approach results in tables and figures in the NFAT submission, MH uses its alternative method of comparing cases.

Given that MH offered an analysis that it now terms "the regrets approach" and featured that analysis prominently in its NFAT Submission, LCA believes it is important to have additional information on this approach to assure that the MH regrets analysis is understood, as well as to reduce the potential for that analysis to provide a distorted and misleading picture of the risk profile. As stated in the cited references to the text in our reports:

> *"By comparing a given plan's result to a base case, the quilt and S Curves are providing a view of "the risk of regret." In other words, would the decision maker look back and regret the choice made if events prove to be different than the reference case assumptions and the investment does not provide the value expected? However, when doing this type of analysis, **it is important to ensure that***

> ***the comparisons being made are, in fact, useful to the decision maker in question.*** *We believe that MH's methodology of comparing all 27 scenarios of a development plan's potential outcomes to a single (reference) point in the base case does not indicate the most important element, which plan is economic for a given scenario."* (Page 9A-61). (emphasis added)

And:

> ***"MH's method of comparing cases and presenting the results is unconventional and can be misleading,*** *as a result. MH' method compares all cases to a single case, All Gas using reference case assumptions on discount rate, energy prices, and capital costs. This does not compare the performance of All Gas to the other plans with comparable sets of assumptions, which we view as more informative and is the more conventional approach to this method of analysis. LCA has developed a model that does the comparative analysis of MH's cases. The LCA Methodology provides a comparative analysis across plans on consistent assumptions of uncertain parameters. Using this method with MH's economic analysis results and probability assessments, the comparative assessments of the plans are quite different than depicted in the MH S-Curve analysis."* (Page 9A-153)

LCA is of the view that a decision of this magnitude does warrant careful analysis of the uncertainties and risks inherent in the decision. The information MH has provided is, at best, incomplete. LCA disagrees with MH's conclusion that some regrets analysis information is helpful, but more regrets analysis information may be misinterpreted.

Further, we note that MH's analysis, and the analysis provided by LCA, is an uncertainty analysis. Neither analysis is a regrets approach to decision making. The proper comparative analysis of alternative development plans under uncertainty can be useful whether a decision is to be made on an expected value criterion, a regrets-based criterion, or any other decision criteria. LCA's analysis provides information that one could use for a regrets-based decision criterion, but our report does not offer any recommendation on such a criterion.

LCA has not made a judgment on how the PUB may choose to weigh the regrets information in its decision making. Rather, LCA is of the view that the PUB should have the benefit of a complete set of information to make that determination.

The foregoing discussion pertains to analysis that focuses on analysis that compares relative risks between alternative development plans. Please also refer to LCA's response to MH/LCA I-076 for a discussion of analysis of the risks inherent within a given plan.

TAB 9

REFERENCE: MIPUG/MH I-009a

QUESTION:

Please provide any references to recent literature about the use or inappropriateness of the "regret approach" in power system planning and evaluation. Please also provide references to the specific analyses and reports prepared and provided publicly by any other the other major hydro utilities in Canada regarding the use or non-use of the regret approach.

RESPONSE:

In general, theories of decision making under uncertainty are either descriptive or prescriptive. Descriptive theories provide formal rules for how individuals (and organizations) actually make decisions; prescriptive theories provide formal rules for how individuals (and organizations) should make decisions.[1] These prescriptive theories are based on fundamental axioms regarding what constitutes rational behavior. For example, transitivity is typically regarded as one essential axiom of rational behavior. Namely, if you prefer A to B and B to C, you really should prefer A to C. For the purposes of NFAT, our focus is on prescriptive decision-making: what should Manitoba Hydro do.

The dominant prescriptive theory is called expected utility. This theory has broad and deep analytical foundations dating back several decades.[2,3,4] Decision analysis, a widely-accepted and well-regarded approach for improved decision-making, is based on expected utility.[5,6] In this context, most forms of scenario analysis, sensitivity analysis, Monte Carlo analysis and the like

[1] There is also a third category: normative. Normative generally refers to how decisions should be made ideally, while prescriptive refers to how they should be made practically.

[2] John von Neumann and Oskar Morgenstern, Theory of Games and Economic Behavior, Princeton University Press, 1944.

[3] P.J.H. Schoemaker, *The Expected Utility Model: Its Variants, Purposes, Evidence and Limitations*, Journal of Economic Literature, 1982.

[4] Peter C. Fishburn, *Analysis of Decisions with Incomplete Knowledge of Probabilities*, Operations Research, March/April 1965.

[5] Ronald A. Howard, *Decision Analysis: Practice and Promise*, Management Science, June 1988.

[6] John Pratt, Howard Raiffa and Robert Schlaifer, Introduction to Statistical Decision Theory, MIT Press, 1995.

can be viewed as variations on decision analysis. Expected utility theory is intended specifically to provide guidance on what constitutes a good decision. Consequently, it is appropriate to use in the NFAT process.

By and large, regret theory is viewed as a descriptive rather than prescriptive theory.[7,8] In fact, it was developed in part in reaction to empirical evidence that individuals (and organizations) do not appear to follow the dictates of expected utility theory; that is, the decisions they actually make are often not the decisions they should make based on axioms of rationality. Regret theory is not generally intended to provide guidance on what constitutes a good decision, and there are very few advocates of regret theory as prescriptive.[9] Even advocates of incorporating regret into a prescriptive theory view it as a limited guideline for individuals not a general guideline for organizational decision making. A quote from David E. Bell sums up this view, "A consumer may wish to spend some...dollars in avoiding disappointment [but this]....paper does not suggest that people ought to make financial tradeoffs to avoid disappointment."[10] Consequently, it is not really appropriate to use in the NFAT process except as a supplement to analysis based more on expected utility.

A simple example will illustrate the difficulties with using the regret approach in the NFAT context.[11] One common axiom of rational decision-making is called "independence of irrelevant alternatives." That is, if Plan A is better than Plan B, Plan B should not suddenly be better than Plan A if a third alternative Plan C is introduced.

[7] Chris Starmer, *Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk*, Journal of Economic Literature, June 2000.
[8] David E. Bell, *Regret in Decision Making Under Uncertainty*, Operations Research, Sep/Oct 1982.
[9] *Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty*, Graham Loomes and Robert Sugden, The Economic Journal, December 1982.
[10] David E. Bell, *Disappointment in Decision Making Under Uncertainty*, Operations Research, Jan/Feb 1985.
[11] *In Praise of the Old Time Religion*, Ronald A. Howard, Utility Theories: Measurements and Applications, 1992.

Table 1 illustrates the choice between two alternatives A and B. The maximum regret for Alternative A is 1 (-50 is 1 worse than -49) and the maximum regret for Alternative B is 50 (50 is 50 worse than 100). Using the standard regret criterion of minimizing the maximum regret, A is better than B.

Table 1: Two Option Example

	Scenario 1	Scenario 2
Alternative A	-50	100
Alternative B	-49	50

Table 2: Three Option Example

	Scenario 1	Scenario 2
Alternative A	-50	100
Alternative B	-49	50
Alternative C	50	-200

Table 2 shows the same problem with a third Alternative C added. Using a regret approach, the maximum regret for Alternative A is now 100 (-50 is 100 worse than 50), the maximum regret for Alternative B is now 99 (-49 is 99 worse than 50), and the maximum regret for Alternative C is 300 (-200 is 300 worse than 100). Based on maximum regret, B is now better than A. With the regret approach, the addition of Alternative C has changed the ranking of A and B. This is generally regarded both by specialists and lay people as illogical, and inappropriate for making good decisions. The bottom line is that, while the regret approach may provide insight into how individuals (and organizations) actually make decisions, it does not provide particularly good guidance on how they should make decisions...particularly organizations.

Empirically, there are many available examples of firms, including electric utilities, using expected utility, or approaches consistent with expected utility, for investment planning and for finding the best investment decision. Nova Scotia's 2009 IRP update includes sensitivity analysis where the range of outcomes associated with each plan is compared.[12] There is no "scenario-by-scenario" regret analysis. In BC Hydro's recent IRP, there is some indication that concepts consistent with regret analysis were considered in the process. However, the main analytical framework and recommendations are based on decision analysis and expected utility.[13]

As far as we can determine, there are no available examples of firms, including electric utilities, using regret theory in any significant way for investment planning and for finding the best investment decision. Bean and Hoppock argue in favor of a regret approach for electric utility planning and use TVA as an example.[14] However, while TVA has mentioned a "least regrets" or "no regrets" approach, its IRP is based on a form of scenario analysis that is more consistent with expected utility.[15] Using more standard terminology, they are actually looking for the "most robust" alternative and their reference to regret really refers to the quality of the decision-making process not the decision itself.

[12] Nova Scotia Utility and Review Board, NSPI 2009 Integrated Resource Plan Update Report, 2009
[13] http://www.bchydro.com/energy-in-bc/meeting_demand_growth/irp/document_centre/reports/november-2013-irp.html
[14] Patrick Bean and David Hoppock, *Least-Risk Planning for Electric Utilities*, Working Paper for Nicholas Institute for Environmental Policy Solutions, August 2013.
[15] http://www.tva.com/irp/pdf/irp_complete.pdf

TAB 10

The role of generalized utility theories in descriptive, prescriptive, and normative decision analysis*

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A number of new theories for decision making under risk have been proposed which relax some properties required by von Neumann–Morgenstern expected utility theory. This paper provides a framework for exploring the usefulness of these theories in the domains of descriptive, prescriptive, and normative decision analysis.

1. Introduction

A number of new theories for decision making under risk relax some requirements of von Neumann–Morgenstern [65] expected utility theory. This paper compares the characteristics of these generalized utility theories and those of expected utility theory and explores the usefulness of these characteristics in the domains of descriptive, prescriptive, and normative decision analysis. An overview of expected utility and generalized utility theories is in Machina [56] and reviews are in Fishburn [31], Machina [55], Sarin [61], and Weber and Camerer [67].

This section briefly discusses the three linked purposes of decision analysis. Section 2 evaluates the performance of expected utility based on normative, prescriptive, and descriptive criteria. Section 3 describes the preliminary evidence on the potential of generalized utility in descriptive decision analysis. Sections 4 and 5 evaluate generalized utility in normative and prescriptive uses. Section 6 presents a summary.

The classification of decision analysis activity into three distinct purposes will highlight some key distinctions among the desirable characteristics in the different domains. Of course, in any one preference theory or decision analysis application there may be a mixture of descriptive, prescriptive, and normative purposes. More discussion on the three purposes is in Brown [8] and Keller [41].

The goal of *normative decision analysis* is to develop models for optimal decision making which have logically, rationally, and morally compelling properties. Sets of compelling properties (or axioms) can be combined to identify various normative preference models, such as von Neumann–Morgenstern expected utility theory for decision making under risk. The appropriateness of these models can then be evaluated on the basis of mathematical correctness, elegance, parsimony, logical coherence, and philosophical arguments in favor of the normative appeal of the properties.

The purpose of *descriptive decision analysis* is to develop models of decision making which provide valid descriptions of actual decision making behavior. Such models can be evaluated on predictive ability, face validity, psychological insights into choice behavior, enhancement of understanding of cognitive processes, elegance.

* A version of this paper was presented at the conference on 'Utility: Theories, Measurements, and Applications', June 1989, in Santa Cruz, CA. The author thanks the conference participants and Henry McMillan for helpful comments.

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parsimony, etc. Much behavioral decision research has focused on identifying deviations of people's actual judgments or choices from those which would result from normative models. (Economists develop *positive* models which are intended to describe the behavior of economic agents, so in this sense such economic models are descriptive. However, economists also examine the validity of their positive models using criteria commonly used to evaluate normative decision models, such as logical coherence and suitability as a besis for further economic-theoretic modeling. Camerer [12] uses the term *productive* to describe the use of models to lay 'individual-level foundations for aggregate-level theory'.)

Prescriptive decision analysis bridges the gap between descriptive observations of the way people do make choices and the normative guidelines for how they should make choices: it prescribes techniques for aiding decision making. For example, finding that business executives participating in an experiment sometimes violated the normatively compelling principle of transitivity of preference orderings, MacCrimmon [57] tested a simple prescriptive technique to overcome these violations. When he verbally pointed out their intransitive orderings, many subjects chose to readjust their orderings and become transitive. Prescriptive decision analysis can be called decision engineering, since analysts are designing techniques to aid decision makers in making better decisions. Analysts must meet design specifications (normative goals), based on descriptive constraints of human judgmental abilities.

Prescriptive decision analysis should be judged with a more holistic focus than that used in evaluating normative decision analysis. As Budge [9] says, researchers should avoid the tendency to test a theory 'to destruction' by sequentially examining separate properties of the theory. Instead, the same theoretical structure should combine logical and ethical considerations with descriptive facts. The whole theory, rather than isolated axioms, should be the unit of analysis. Thus, the existence of appropriate and easily implemented methods for assessing preferences and a feasible analysis procedure will be just as important as logical coherence when a theory is used for prescriptive purposes. In addition, a prescriptive theory should take into account descriptive research to adjust for biased judgments of preferences, probabilities, or problem structure received from decision makers. Such a theory should also be designed to operate on the set of data available. For example, good choices should be prescribed even if data on alternative actions or states of nature are incomplete. Brown [8] believes 'that the validation of prescriptive technology should be primarily external, i.e., tools . . . (should be) tested by confrontation with the outside world', rather than focusing on tests of the internal logical consistency of models. He proposes five criteria for evaluating a prescriptive technique: technical soundness (the technique appropriately uses all relevant information and judgment), cost of delaying the decision while the technique is used, cognitive burden on the decision maker, acceptability to institution, and psychological acceptability.

Brown [8] expands the conception of 'prescriptive' to include developing decision making aids for more realistic decision environments. Normative models often assume the decision maker has complete knowledge of all the alternative actions, states of nature, payoffs and probabilities at a single point in time, at which the decision (or set of contingent, sequential decision plans) will be made. A broader conception of the modeling environment may lead, for example, to a focus on better problem structuring or to models allowing sequential restructuring of the problem. Keller and Ho's [45] work on problem structuring is an example of research which is in this very different but equally important genre. This broader notion of the proper role of prescriptive decision analysis will not be addressed further in this paper.

In light of the threefold purposes of decision analysis, a central question is to what extent researchers and decision technologists should attempt to use a single theory for dual or triple purposes. Much of the debate about the new generalized utility theories' characteristics can be clarified by clearly specifying the purposes of the proposed uses of the theories. The suitability of a theory should then be judged by the appropriate criteria for the chosen purpose(s).

2. The performance of expected utility in normative, prescriptive, and descriptive decision analysis

Expected utility theory is probably the most widely accepted normative theory for decision making under risk. Von Neumann and Morgenstern [65] axiomatized expected utility theory by showing that, if a set of apparently normatively appealing axioms hold, alternative actions can be ranked by their expected utilities. The expected utility is a weighted average of the utilities of the possible outcomes where the weights are the objective probabilities of each outcome. Savage's subjective expected utility model allows the derivation of a decision maker's own subjective probabilities for events, which are then used to compute the subjective expected utility of each alternative. Edwards [28, 29] and other psychologists have experimentally investigated a model wherein a person makes choices as if he or she transforms the objective probabilities into subjective probabilities, then computes expected utility via the resulting subjective probability weighting function.

2.1. Normative decision analysis and expected utility

Expected utility theory's original and primary purpose is normative. It is elegant, logically coherent, and parsimonious. It requires preference information from a person for only a few choices to identify a utility function which can be used to specify the normatively 'correct' choices for all possible related choice situations. The preference principles characterizing expected utility are generally compelling, although there is debate over whether the substitution (independence) axiom should hold normatively.

The *substitution principle* of expected utility theory requires that whenever some lottery A is preferred or indifferent to a lottery B, then the compound lottery $pA + (1-p)Z$ must be preferred or indifferent to the compound lottery $pB + (1-p)Z$. The compound lottery $pA + (1-p)Z$ is formed by having a p chance of getting lottery A and a $(1-p)$ chance of getting lottery Z, for any probability values p ranging from 0 to 1. So, a decision maker who prefers the sure $3200 in option A in Fig. 1 over the risky option B also must prefer D over E, since D and E are formed by substituting lotteries A and B, respectively, into an otherwise identical lottery with a 10% chance of A or B and a 90% chance of Z (where Z is the degenerate lottery of getting $0 for sure). Most people choose A over B and E over D. This most common response pattern violates the substitution principle, and thus expected utility, as will be discussed later in Section 2.3 on the descriptive usefulness of expected utility.

In a dynamic setting, expected utility theory has the property of *dynamic consistency*, i.e., if a person has option C at time 0 in Fig. 1, the planned choice between A' and B' made at time 0 should agree with the actual choice made at time 1. Notice that the planned choice of CA' (C then A') is strategically equivalent to D and the choice of CB' is equivalent to E [54]. By the substitution principle, if the actual choice is A' over B', then D is preferred over E, so the planned choice will be CA' over CB'.

Expected utility is *linear in probabilities*, since the expected utility

$$EU(pA + (1-p)B) = pEU(A) + (1-p)EU(B).$$

For this reason, it is sometimes called *linear expected utility*. In a Marschak triangle diagram graphically representing the set of all possible alternative actions with probability distributions over three fixed outcomes (see, e.g., [56]), this means that indifference curves are linear and parallel. Expected utility preferences are *separable* across mutually exclusive events, in the sense of *replacement separability* (the contribution of each outcome x_i and its probability p_i to the overall expected utility of an alternative action is independent of the other outcome/probability pairs) and *mixture separability* (the contribution of each outcome/probability pair to the overall expected utility can be broken down into the utility of x_i, multiplied by p_i).

Expected utility also satisfies *consequentialism* [54]. At any point in time we can focus on the consequences from now on (choices, states, probabilities, and outcomes) and we do not need

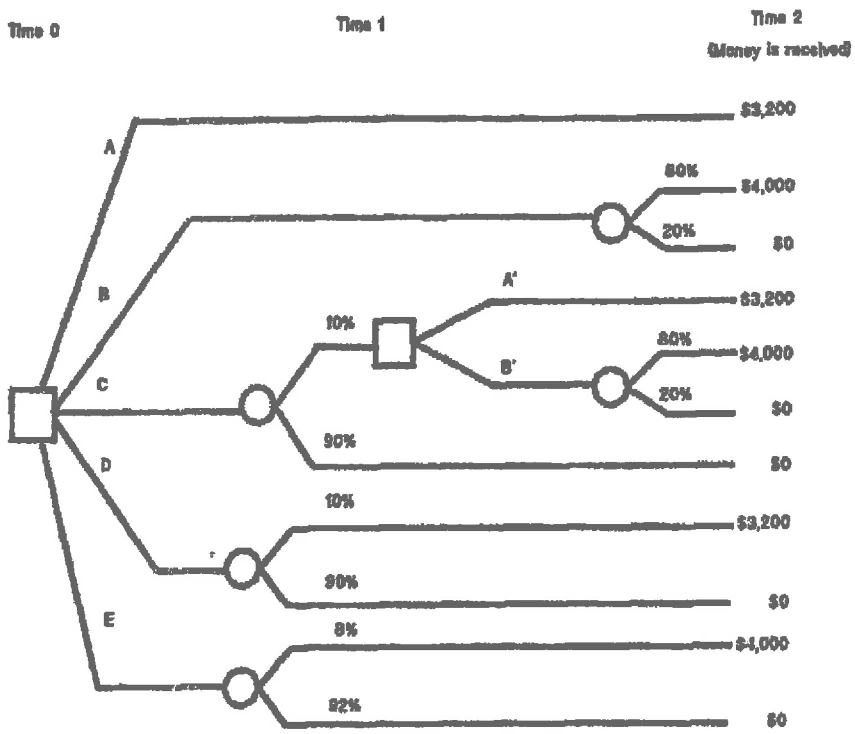

Fig. 1. Decision tree.

to know where we have come from or what other probability or choice branches were previously available. Thus, the analysis of the expected utility of alternative actions can be carried out by '*folding back*' a decision tree representation of the choices and states, by computing the maximum expected utility based on the options and states remaining at any one point in time.

Additional characteristics of expected utility include the transitivity, common-consequence (sure-thing), reduction-of-compound-lotteries, and betweenness principles. Expected utility orderings are consistent with first-order stochastic dominance rankings. Finally, expected utility requires ambiguity indifference (i.e., indifference between two risky options which are identical except that one option has a non-vague subjective probability p for an event and the other has the same subjective probability for a corresponding event, but the probability p is ambiguous; see [30]).

A question not directly addressed by utility theory is the choice of risk attitude. Under expected utility theory, a person is labeled risk averse if a sure monetary amount (such as the $3200 in Option A in Fig. 1) is preferred over a lottery (such as Option B) whose expected monetary value is equal to that sure amount. This labeling scheme is misleading, because it mixes attitude towards risk with strength of preference for different outcomes. An unresolved question is whether risk attitude should be a by-product of assessment judgments (as it is in utility theory assessment procedures) or it should be a conscious decision. For example, a person might choose to be relatively risk neutral [27, 44] over a certain range of outcomes. Utility theory can accommodate either approach, since only the assessment procedures need to be modified to guarantee a specific risk attitude prior to the calibration of the utility function. A related question is whether an S-shaped utility function, with a point of inflection at a target or reference level should be allowed in normative or prescriptive decision analysis. Some people (especially economists) argue that a person should retain either risk aversion, proneness, or neutrality over all outcome domains.

2.2. Prescriptive decision analysis and expected utility

Expected utility is well-suited as a prescriptive model for decision making under risk. First, its strong normative appeal has made it popular among decision analysts. Second, the required analysis is straightforward, assuming all required information on alternative actions, states, probabilities, and outcomes are available. Decision trees can be constructed by hand or with existing computer software (such as Arborist, Supertree, etc.) and then analyzed by folding back the tree. Third, multiattribute utility models exist [40], so a more accurate description of problem attributes is possible. Fourth, many applications have already been successfully carried out, so the theory has been field tested. Reviews of applications are in Keeney and Raiffa [40], Howard et al. [38], and von Winterfeldt and Edwards [66].

A primary problem with expected utility as a prescriptive decision analysis technique is *assessment of the utility function*. Assessed functions can often vary systematically with the response method. In a common assessment procedure, the decision maker adjusts a sure monetary amount to determine the *certainty equivalent* which attains indifference with a lottery. This method can result in a different utility function than the one the same person would get if a probability is adjusted to determine a *probability equivalent* which matches one option to an indifferent option [36, 37]. Further, certainty equivalents may vary depending on the way they are assessed. In the Becker–deGroot–Marschak [2] mechanism for promoting 'correct' certainty equivalent responses, the experimenter offers to buy the lottery from the subjects if a randomly generated offer price exceeds their stated minimum selling price. In preliminary experimental work, Uzi Segal and I have found that assessment of certainty equivalents for a lottery via this mechanism can lead to different certainty equivalent values (when the random offer price is drawn from a larger range of possible prices), as implied by at least one generalized utility theory [60].

Since people often violate the substitution principle, another assessment problem can occur. For example, in the Fig. 1 choice situation, people often appear risk averse by choosing option *A* to get a sure $3200 over the risky option *B* which has an 80% chance of $4000 (or else $0). The expected utility function assessed with such question responses is concave, reflecting risk aversion. However, if expected utility is assessed over the same range with questions containing low probabilities for the positive, non-zero outcomes, such as *D versus E* in the figure, the function would be convex, reflecting risk proneness, if *E* is preferred over *D*. Such a preference for *A* and *E* violates the substitution principle of expected utility. The problem for prescriptive decision analysis is not that expected utility is violated, since the analysis process of applying an assessed utility function to a problem will guarantee that the substitution principle and expected utility are obeyed. Rather, the problem is that assessment questions, which by expected utility standards should yield identical utility functions, can produce widely varying utility functions, which may even switch from risk aversion to risk proneness.

Another assessment problem is the discrepancy between preferences when elicited with paired comparison and direct rating methods. This discrepancy is called the *preference reversal phenomenon* [49, 35]. Although more research needs to be done on the effects of response modes on expressed preferences to support practical use of preference assessment technologies, two approaches seem promising. First, Tversky et al. [64] have introduced a contingent weighting model to represent the difference in inferred preferences resulting from different response modes. Also, Bostic et al. [7] found that a choice-based sequential procedure for discovering certainty equivalents holds promise for eliminating the systematic overstating of the value for lotteries with a moderate probability of a large gain.

Another issue in prescriptive decision analysis is determining *how far to aid a decision maker* in restructuring the problem and the relevant preferences. If a person's subjective probability of an injury when not wearing seat belts is too low, seat belts might not be worn when they 'should' be, based on the objective probability of injury. In this case, an analyst may wish to point out objective probability information. Similarly, a person may violate principles required by expected utility. An analyst is confronted with deciding how far to push a person to conform with expected utility before allowing the person to violate these principles and use a generalized utility model for guiding choice. Also, altering the framing of a decision situation by adjusting the perceived status quo (or reference) level can greatly alter a decision maker's perspective and risk taking behavior. A reasonable prescription is to limit the number of times a decision maker resets the reference level, as suggested by von Winterfeldt and Edwards [66, pp. 373–377].

2.3. *Descriptive decision analysis and expected utility*

The descriptive validity of expected utility has been strongly challenged in two types of laboratory experiments. Most experiments have examined patterns of choices to demonstrate violations of expected utility principles. A few recent experiments have gone further and actually assessed subjects' expected utility to determine the percentage of choices correctly predicted.

First, a fairly large body of experimental evidence shows that subjects systematically make choices which violate principles required by expected utility. Substitution (or common-ratio or independence) principle and sure-thing (or common consequence) principle violations have been shown by, e.g., MacCrimmon and Larsson [58], Kahneman and Tversky [39], and Keller [42]. Violations of the reduction of compound lotteries principle [43] and the betweenness principle [20, 21, 22] have also been shown. Aversion to ambiguity in probabilities [30] has also been demonstrated in experiments and models have been proposed to accommodate non-indifference to ambiguous probabilities, but they will not be addressed in this paper.

Second, preliminary evidence shows that assessed and/or fitted expected utility functions predict choices moderately well, but with much room for improvement. Currim and Sarin [23] assessed experimental subjects' expected utility and prospect theory models, and Daniels and Keller [24] assessed expected utility and lottery dependent utility models. In both cases, expected utility did about as well as the two generalized utility models in predicting choices on a hold-out sample of paired comparison choices, even when the problems were structured to induce substitution or sure-thing principle violations.

When examining predictive performance it is important to examine how much utility is lost by using a specific model. A model may not accurately predict all choices, but it may predict the correct choices whenever there is a big difference in the perceived value of the two options. For example, Daniels and Keller [24] calculated the utility difference between predicted and actual choices in addition to tabulating the number of correct predictions by a model. Future experimental work should measure the magnitude of the potential mistakes in prediction resulting from a specific model.

Part of the appeal of the expected utility model is its simplicity, with preference defined on the probability distribution over outcomes. Actual decision making depends on many additional factors, such as fear, regret, context, memory capacity, processing capacity, and framing effects. Perhaps entirely different models should be used for describing choice. An important criterion for evaluating descriptive research is the potential for insights on behavior coming from the model. A risk averse expected utility function may say something about a person's psychological attitude toward risk, but it is confounded with strength of preference. A model which may provide more insight about peoples' thought processes, for example, is Lopes' [51] two-factor theory for risky choice, which combines a dispositional factor (desire for security versus potential) with an aspiration level factor.

There are at least two different categories of responses to the descriptive violations of expected utility. One response, followed in Keller [42, 43] is to develop prescriptive techniques, such as visual problem representations, to aid decision makers to conform with expected utility theory. The other response is to develop new descriptive models which are empirically valid. Some will say that the generalized expected utility models which relax the substitution principle, or other principles, are designed for the purpose of creating a descriptively valid model. If so, then the generalized models should be judged by the entire set of criteria used for descriptive decision analysis. Others may argue that there is a combination of descriptive and normative purposes behind the new generalizations of utility theory, and they should be judged both descriptively and normatively. Still others argue that *if* generalized utility theories are normatively appropriate, then perhaps they should be used for prescribing decisions, and they should then be evaluated on the basis of prescriptive criteria. The next three sections examine the properties of the new generalizations of expected utility from the perspective of one of the three purposes of decision analysis. Potential users of the generalized utility technology should take into account the criteria and issues to be discussed when choosing among the existing or still-to-be developed preference modeling approaches.

3. The performance of generalized utility in descriptive decision analysis

Many generalized utility theories have been recently proposed as variants of expected utility theory. Some of these theories include prospect theory [39]; weighted utility [13, 17, 18] and the related skew-symmetric bilinear utility [32, 33] and regret theory [6, 50]; lottery dependent utility [4]; approximate expected utility [48]; expected utility with rank dependent probabilities (Quiggin's [59] anticipated utility, Yaari [68], Luce and Narens' [52] dual bilinear utility); general quadratic utility [16, 53, n. 45]; implicit expected utility [14, 25]; and ordinal independence [62, 34].

Since their development was primarily motivated by descriptive violations of expected utility theory principles, most generalized theories are designed to account for these violations. Thus, they generally have the potential to describe choices which have been observed in laboratory settings. This potential is usually first demonstrated theoretically by showing the model is mathematically able to match non-expected utility choices. Next, new data are collected for existing or new questions to show the preference patterns the new models are theoretically capable of predicting; e.g., Chew and Waller [19] followed this approach to evaluate weighted utility theory.

Camerer [12] contrasted several generalized utility theories on the basis of implied preference patterns and collected experimental subjects' choices. The theories included weighted utility, and the related skew-symmetric bilinear utility and regret theory; implicit expected utility theory; the fanning-out hypothesis of Machina [53]; lottery dependent expected utility; prospect theory; and expected utility with rank dependent probabilities. Camerer examined sets of choices to gather evidence on subjects' indifference curves. Indifference curves which are parallel straight lines in the Marschak triangle conform with expected utility, and non-parallel indifference curves violate expected utility. The predominant patterns of choices violating the substitution and sure-thing principles can be represented by preference models which allow indifference curves to fan out. Fanning-in indifference curves correspond to violations of the common consequence principle (and thus expected utility), but not in the most common way. Camerer found evidence of both fanning-out and fanning-in of indifference curves. No one existing theory could explain all the preference data, but prospect theory and the fanning-out hypothesis matched most of the data.

Although the generalized theories often have the potential, in principle, to match non-expected utility choices, for successful use as a descriptive model to predict choice, the predictive performance must also be examined. This has yet to be attempted for the majority of the models. In fact, many models still do not have a precise enough form that a preference function can be assessed or fitted so predictions on choices between arbitrary options can be made. For example, Leland [48] posits the existence of an approximate utility function with a step function form, so that sometimes two close outcomes are identical in 'approximate' utility, but he does not show how to assess such a function. Currim and

Sarin [23] have elicited prospect theory functions and Daniels and Keller [24] elicited lottery dependent utility theory functions. These first investigations showed that the two generalized models did about the same as expected utility in predicting choices when assessed with the usual certainty or probability equivalent procedures, but that there was much room for improvement if specialized assessment procedures could be developed. The generalized models did predict patterns of preferences violating expected utility, but they did not always predict the specific choices for a subject. Further, the problem of variation in assessed functions arising due to different response modes (which was discussed under prescriptive expected utility) remains unsolved.

It is important to note that descriptive models may not need to be assessed to meet certain uses. For example, economists, including Machina and Leland, have developed theoretical generalized utility models whose general properties can be used in further economic-theoretic modeling without specifying a precise functional form. However, in other cases, the theory must be precise enough to allow prediction of choices among any set of arbitrarily chosen options. For example, suppose a firm wants to predict the market share for alternative warranties for consumer durables. Using a theory facilitating preference assessment, survey respondents could answer simple questions to assess their preference functions, which could then be used to predict their choices among the alternative warranty policies. In the latter case, a model, such as expected utility, may be preferred because of good predictive performance and ease of data collection even if it requires preference properties, such as the substitution principle, which are known to be descriptively violated. Thus, expected utility theory should not be ruled out as a useful predictive theory until it is replaced with a theory that clearly does better in predicting arbitrary choices.

The generalized utility theories share with expected utility the advantage and disadvantage of requiring only information on the probability distribution over outcomes and on a person's preference judgments. Such models may not capture the richness needed in operational contexts. Prospect theory does enrich the domain of the model by adding a preliminary stage in which a problem is framed and encoded for subsequent analysis, but the specifics of this preliminary stage need to be further developed.

4. The performance of generalized utility in normative decision analysis

The potential performance of generalized utility in normative decision analysis will be determined on the basis of the normative acceptability of the characteristics of various non-expected utility models. The characteristics of expected utility will generally be used as a baseline for the philosophical debate on normatively desired characteristics. For example, the original version of prospect theory may violate the normatively compelling property of *first-order stochastic dominance preference* [54, footnote 17], which is satisfied by expected utility and some generalized utility models. (A new rank-dependent form of prospect theory is under development which does not violate stochastic dominance.) This section contains the normative arguments for and against different characteristics. Of special concern when *substitution principle violations for static lotteries are allowed*, is whether *dynamic* consistency and/or *consequentialism* should hold.

Machina [54] argues that non-expected utility models which would be used in economic theory should have the arguably normative properties of dynamic consistency and non-consequentialism. (See also Chew and Epstein [15].) Non-consequentialism means that the choice between A' and B' at time 1 in Fig. 1 cannot be made without knowing that there was a previous 10% probability of arriving at the choice node at time 1, and a 90% probability of the outcome $0 which might have happened had Option C been chosen at time 0. Such a dynamically consistent non-expected utility model would not always obey the substitution principle applied to static single stage lotteries, and could thus model the simultaneous preference among single stage lotteries of A over B but E over D in the figure. However, using a dynamically consistent non-expected utility model, under option C the planned choice between A' and B' at time 0 in the decision tree in Fig. 1 would have to agree with the actual choice made at time 1. A decision maker with these preferences would be classified as a *gamma-type* according to Machina's [54] categorization of decision makers into *alpha*, *beta*, *gamma*, and *delta* types, as shown in Fig. 2. *Alpha-types* use expected utility and thus obey the substitution principle, consequentialism, and dynamic consistency. *Betas*, *gammas*, *deltas* (and an added type: *epsilons*) sometimes violate the substitution principle for static lotteries.

Machina is concerned that economic researchers will not accept a model which can potentially predict dynamically inconsistent choices. This behavior arises by being a consequentialist and isolating the focus at time 1 only on A' and B', perhaps choosing A' over B', having planned on CB' over CA' originally. The argument against dynamic inconsistency is normative. It hinges on the possibility that a person can be made to 'make book' against his/her own choices, making the person into a perpetual money pump, cycling among options to eventual ruin. Adding to this normative argument the descriptive observation that such money pumps are not observed in economic markets, Machina [54] rejects dynamic inconsistency. Thus, he rejects *beta-type* preferences (consequentialist, not dynamically consistent, substitution principle violators) and, implicitly, *epsilon-type* preferences (which only differ from betas on not being consequentialists).

As an aside, it seems that a better approach to economic modeling, due to the need for descriptive validity, would be to continue the search for mathematically tractable theories which are descriptively valid, both for individual judgment behavior and for the observed aggregate market behavior. First, examination of the market behavior may reveal isolated judgments which can be shown to be dynamically inconsistent. For example, perhaps money pumps have not been found because our model of the decision situation is too simplified. Since people do not purchase houses frequently, it would be difficult to observe a person repeatedly buying and selling houses, cycling down to eventual ruin. But, it may be possible to find a person isolating the house selling problem at the current stage (to sell or rent) rather than recalling the previous probability of getting to the stage of having to move and the other previous probabilistic branches. Such a person may choose the option of selling (which may reflect risk aversion if the selling price is a sure amount), even though the planned choice three years previously (prior to gaining information about the need to move to a new job) might have been to rent (if and when a move had to be made). Renting may have a wider distribution of possible income flows depending on the renter availability and possible damages, and thus may be a risk prone choice.

I believe that non-expected utility models were developed in response to both types of substitution principle violations, those for static choices and for dynamic (multiple-stage) choices. Since experimental evidence suggests that this is how people see the problem and make their choices, a descriptively valid model of decision making under risk should definitely allow the planned choice to differ from the actual choice. Further, a good argument can be made that a normative model should allow the difference between planned and actual choices if a reasonable decision maker chooses, upon reflection, to make different choices. Sarin [61] presents the philosophical debate over whether dynamic consistency should hold in normative models and argues that a

	Dynamic consistency: Inconsistent $CA' \le CB'$ and $A' > B'$ occurs	Dynamic consistency: Consistent $CA' > CB' \leftrightarrow A' < B'$
Consequentialist $A > B \leftrightarrow A' > B'$	Beta	Delta
Not consequentialist $A \le B$ and $A' > B'$ occurs	Epsilon	Gamma

Fig. 2. Classification of decision makers who violate substitution principle for static lotteries ($A > B$ and $D \le E$ occurs). (Notes: $>$ and $\le$ indicate preference order; A, A', B, B', C, D, and E are options in Fig. 1; Alpha-type (expected utility) preferences obey substitution principle, consequentialism, and dynamic consistency.)

decision maker may wish to violate dynamic consistency.

For example, the lottery dependent utility theory of Becker and Sarin [4] will allow planned choices to differ from actual. Applying their model to option C in Fig. 1's decision tree problem, at time 0, a *beta-type* consequentialist who is not dynamically consistent might note that CB' is strategically equivalent to E and choose the *planned choice* CB' over CA', which is strategically equivalent to D. Then, whenever the decision node at time 1 arises, this consequentialist beta-type revises the tree and only compares A' and B', and may choose A' as the *actual choice*.

Whether a particular generalized model represents dynamically consistent choices may depend not on the model per se, but on the way it is applied to choice situations and how the decision maker frames and reframes choices over time. For example, Becker and Sarin [5] show how to analyze the utility of alternatives using a modified approach for folding back a decision tree. Following this analysis procedure yields *delta-type* preferences which are dynamically consistent since planned choice equals actual choice, because plans are always made by working backwards through the entire tree. This procedure is also consequentialist, since folding back the decision tree to determine choice is done by isolating focus on the current and future stages only. Machina [54] presents three arguments against delta-type people: (1) strategically equivalent lotteries will not be indifferent [46, 47]; (2) aversion to costless information in decision trees, and (3) folding 'back is only appropriate when the objective function is separable across the various subdecisions of a problem'. Further debate should be conducted on the merits of dynamic consistency and consequentialism as normative principles, starting with an investigation of these three criticisms.

5. The performance of generalized utility in prescriptive decision analysis

If generalized utility models are to be used in aiding decision making, they should be *assessable*, have *feasible analysis procedures*, and recommend *problem framings with face validity*, i.e., problem framings which are acceptable to the decision maker. These practical concerns should be weighed against the concerns of the normative appropriateness of the models, which were discussed above.

Models relaxing the substitution principle need not be harder to assess. Currim and Sarin [23] have shown how to assess prospect theory in an experimental comparison of the predictive performance of prospect theory and expected utility. Keller and Daniels [24] have assessed lottery dependent expected utility theory of Becker and Sarin [4] and expected utility theory. In both cases expected utility theory did about as well as the generalized theories.

However, if very complicated mathematical forms are needed, if assessment questions are too lengthy or complicated, or if different functions must be assessed in local regions, a generalized utility model may not be practical for actual applications. Further, with the level of assessment precision attainable with our current assessment procedures, theoretically different models may differ little in their prescribed choices. For example, a specific decision maker's concave expected utility function assessed with an exponential form and with a power form may be indistinguishable over the range of relevant outcomes and for the attainable level of assessment precision, even though the two functional forms imply different preference attitudes. More research needs to be done to determine the likely difference between generalized utility models in prescriptive performance.

The feasibility of the analysis of generalized utility models requires that an assessed or fitted model can be obtained, that sufficient data on the problem structure (probabilities, states, alternatives, and outcomes) is available, and that computation of maximum utility is possible (by hand or computer). Some generalized utility models are criticized because their analysis procedure does not allow folding back the decision tree as is possible under expected utility, which is consequentialist and focuses only on the current and future time periods to make a choice among current options [47]. However, as described earlier, Becker and Sarin [5] show how their generalized utility model, lottery dependent utility, can be used in a modified folding back procedure. Further, with the availability of computers, the criterion of 'ease of hand calculation' can now be replaced by the 'availability of computer programmable computational procedures', so not being able to fold back a tree is not an insurmountable analytic problem.

Since the choice of the way to frame the current decision problem can alter the choice prescribed by generalized utility models, framing issues are of considerable practical concern. Should you frame your life decisions as being at the actual current decision point or at the initial life planning point (say at age 12)? If you model your problem as if you are at the initial point you may plan to be risk prone with respect to the choice of a job after college graduation, but at the actual job choice point you may be risk averse. Also, when to stop elaborating the decision tree into the future [46] must be decided. Some will argue that these framing problems should be avoided by retaining the expected utility model for prescriptive decision analysis. This requires, for example, that at time 0 in Fig. 1, if A is better than B, then D is better than E, and CA' is better than CB'; and at time 1, A' is better than B'. More philosophical debate is needed to resolve the issue of which models are prescriptively useful, given that different models' choice prescriptions remain invariant among different sets of transformations of problem frames.

I am now inclined to allow decision makers to choose among possible problem structures, after showing them the alternative framings which are supposedly equivalent (from an expected utility perspective). So, if a house seller perceives the current decision situation as a choice between sell or rent and chooses to ignore previous branches in the decision tree, allow it. Then aid choice at that point, possibly with a generalized utility model, even if the choice (of, say, the risk averse option of selling) may be seen as dynamically inconsistent if framed in the context of a five-year planning period, starting three years previously. Back then, since the chance of ending up at this decision node was probably low, the planned choice might have been the risk prone choice of renting. Such an approach allows the decision maker to identify the psychologically relevant problem structure for the current problem. This approach must be contrasted with one of the alternatives, which is to alter the person's world view sufficiently so that all current choices are seen as only one of the myriad of possible choices which could have been presented, rather than the relatively circumscribed pseudo-certain frame of a current choice that is probably more common. The alternative approach edges on altering culturally-based perceptions of time and fate, and should be examined carefully.

6. Summary

This paper has provided a framework for examining the potential of generalized utility theories for use in descriptive, normative, and prescriptive decision analysis. The decision analysis community should evaluate the relative value of alternative directions for future research and technology development in meeting these three linked purposes for decision analysis.

A major gap now exists in which the many models which can potentially represent non-expected utility preferences have yet to be evaluated on the basis of predictive performance. Further, more philosophical debate is needed to resolve the issue of the allowable transformations of problem frames which yield identical prescriptions by preference models.

Also, behavioral decision research has up to now focused on investigating deviations from normative models. An entirely new paradigm, containing psychologically relevant variables such as disposition to seek security, and variables which can actually be controlled in the decision environment may yield more benefits for understanding how people make unaided decisions. For example, in medical decision making, a descriptive model could include format, context, and interaction with medical professionals to represent patients' choices between risky surgical and medical alternatives.

References

[1] M. Allais, Le comportemente de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école americaine, Econometrica 21 (1953) 503–546.

[2] Gordon Becker, Morris deGroot and Jacob Marschak, Measuring utility by a single-response sequential method. Behavioral Sci. 9 (1964) 226–232.

[3] Joao Luiz Becker. A new model of decisions under risk using the concept of lottery dependent utility function. Unpublished doctoral dissertation, UCLA, Graduate School of Management (1986).

[4] Joao Becker and Rakesh Sarin. Lottery dependent utility. Management Sci. 33(11) (1987) 1367–1382.

[5] Joao Becker and Rakesh Sarin. Decision analysis using lottery dependent utility. J. Risk and Uncertainty 2(1) (1989) 105–117.

[6] David Bell. Regret in decision making under uncertainty. Operat. Res. 30 (1982) 961–981.

[7] Raphael Bostic, R.J. Herrnstein and R. Duncan Luce. The effect on the preference-reversal phenomenon of using choice indifferences. Working paper (Herrnstein). Dept. of Psychology, Harvard University, 33 Kirkland St., Cambridge, Massachusetts 02138 (1989).

[8] Rex Brown. Toward a prescriptive science and technology of decision-aiding. Ann. Operat. Res. 19 (1989).

[9] Ian Budge. Benthamism as a 'positive' rational choice theory of democracy: Does it work? In: Lincoln Allison, ed., The Utilitarian Response: Essays on the Contemporary Viability of Utilitarianism as a Political Philosophy (Sage, London, forthcoming) chapter X.

[10] J. Greg Byrd, Richard deNeufville and Philippe Delquie. The difference between probability and certainty equivalent methods of utility assessment. Working paper, Technology and Policy Program, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 (1987).

[11] Colin Camerer. An experimental test of several generalized utility theories. J. Risk and Uncertainty 2(1) (1989) 61–104.

[12] Colin Camerer. Generalizations of expected utility theory: the latest evidence. Working paper, Wharton, University of Pennsylvania (1989).

[13] Chew Soo Hong. A generalization of the quasilinear mean with applications to the measurement of income inequality and decision theory resolving the Allais paradox. Econometrica 51 (1983) 1065–1092.

[14] Chew Soo Hong. Implicit-weighted and semi-weighted utility theories, *M*-estimators, and non-demand revelation of second-price auctions for an uncertain auctioned object. Working paper #155, Johns Hopkins University, Department of Political Economy (1985).

[15] Chew Soo Hong and Larry Epstein. Non-expected utility preferences in a temporal framework with an application to consumption-savings behaviour. Working paper, Johns Hopkins, Department of Political Economy, Baltimore, Maryland 21218 (Chew) (1989).

[16] Chew Soo Hong, Larry Epstein and Uzi Segal. Mixture symmetric utility theory. Working paper, University of Toronto (1988).

[17] Chew Soo Hong and K.R. MacCrimmon. Alpha-nu choice theory: a generalization of expected utility theory. Working paper no. 669, University of British Columbia, Faculty of Commerce and Business Administration, Vancouver (1979a).

[18] Chew Soo Hong and K.R. MacCrimmon. Alpha utility theory, lottery composition and the Allais paradox. Working paper no. 686, University of British Columbia, Faculty of Commerce and Business Administration, Vancouver (1979b).

[19] Chew Soo Hong and William S. Waller. Empirical tests of weighted utility theory. J. Math. Psych. 30 (1986) 55–72.

[20] Clyde Coombs. Portfolio theory: A theory of risky decision making. La Decision (Centre National de la Recherche Scientifique, Paris, 1969).

[21] Clyde Coombs. Portfolio theory and the measurement of risk. In: Martin Kaplan and Steven Schwartz, eds., Human Judgment and Decision Processes (Academic Press, New York, 1975) pp. 63–85.

[22] Clyde Coombs and Lily Huang. Tests of a portfolio theory of risk preference. J. Exp. Psych. 85(1) (1970) 23–29.

[23] Imran Currim and Rakesh Sarin. Prospect versus utility. Management Sci. 35(1) (1989) 22–41.

[24] Richard Daniels and L. Robin Keller. An experimental evaluation of the descriptive validity of lottery dependent utility theory. J. Risk and Uncertainty (forthcoming 1990).

[25] Eddie Dekel. An axiomatic characterization of preferences under uncertainty: weakening the independence axiom. J. Econ. Theory 40 (1986) 304–318.

[26] Philippe Delquie, Richard de Neufville and Herve Mangan. Response-mode effects in preference elicitation. Working paper, Technology and Policy Program, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 (1987).

[27] James S. Dyer and Rakesh K. Sarin. Relative risk aversion. Management Sci. 28(8) (1982) 875–886.

[28] Ward Edwards. The prediction of decisions among bets. J. Exp. Psych. 50 (1955) 201–214.

[29] Ward Edwards. Subjective probabilities inferred from decisions. Psych. Rev. 69 (1962) 109–135.

[30] Daniel Ellsberg. Risk, ambiguity, and the Savage axioms. Quart. J. Econ. 75 (1961) 643–669.

[31] Peter C. Fishburn. Foundations of decision analysis: along the way. Management Sci. 35(4) (1989) 387–405.

[32] Peter C. Fishburn. Transitive measurable utility. J. Econ. Theory 31 (1983) 293–317.

[33] Peter C. Fishburn. SSB utility theory: an economic perspective. Math. Social Sci. 8 (1984) 63–94.

[34] J. Green and B. Jullien. Ordinal independence in nonlinear utility theory. J. Risk Uncertainty 1(4) (1988) 355–387.

[35] David M. Grether and Charles P. Plott. Economic theory of choice and the preference reversal phenomenon. Amer. Econ. Rev. 69 (4) (1979) 623–638.

[36] J.C. Hershey, Howard Kunreuther and Paul Schoemaker. Sources of bias in assessment procedures for utility functions. Management Sci. 28(8) (1982) 936–954.

[37] J. Hershey and Paul Schoemaker. Probability versus certainty equivalence methods in utility measurement: Are they equivalent? Management Sci. 31 (1985) 1213–1231.

[38] Ron A. Howard, J.E. Matheson and R.L. Miller, eds., Readings in Decision Analysis (Stanford Research Institute, Menlo Park, CA, 1976).

[39] Daniel Kahneman and Amos Tversky, Prospect theory: an analysis of decision under risk. Econometrica 47(2) (1979) 263–291.

[40] Ralph L. Keeney and Howard Raiffa, Decisions with Multiple Objectives: Preferences and Value Tradeoffs (John Wiley, New York, 1976).

[41] L. Robin Keller, Decision research with descriptive, prescriptive, and normative purposes – Some comments. Ann. Oper. Res. 19 (1989) 485–487, volume edited by I. LaValle and P. Fishburn, on Choice Under Uncertainty.

[42] L. Robin Keller, The effects of problem representation on the sure-thing and substitution principles. Management Sci. 31(6) (1985a) 738–751.

[43] L. Robin Keller, Testing the 'reduction of compound alternatives' principle. OMEGA, The Int. J. Management Sci. 13(4) (1985b) 349–358.

[44] L. Robin Keller, An empirical investigation of relative risk aversion. IEEE Trans. Systems, Man, and Cybernetics 15(4) (1985c) 475–482.

[45] L. Robin Keller and Joanna Ho, Decision problem structuring: Generating options. IEEE Trans. on Systems, Man, and Cybernetics 18(5) (1988) 715–728.

[46] Irving H. LaValle, New choice models raise new difficulties: comment on 'Analytical issues in decision methodology'. In: Ira Horowitz, ed., Decision and Organizational Theory (Kluwer-Nijhoff, Dordrecht, 1989).

[47] Irving H. LaValle and Kenneth R. Wapman, Rolling back decision trees requires the independence axiom! Management Sci. 32 (1986) 382–385.

[48] Jonathan Leland, A theory of 'approximate' expected utility maximization. Working paper, Carnegie Mellon University (1988).

[49] Sarah Lichtenstein and Paul Slovic, Reversals of preference between bids and choices in gambling decisions. J. Exp. Psych. 89 (1) (1971) 46–55.

[50] Graham Loomes and R. Sugden, Regret theory: an alternative theory of rational choice under uncertainty. Econ. J. 92 (1982) 805–824.

[51] Lola L. Lopes, Between hope and fear: the psychology of risk. Adv. Exp. Psych. (1987).

[52] R. Duncan Luce and Louis Narens, Classification of concatenation structures according to scale type. J. Math. Psych. 29 (1989) 1–72.

[53] Mark Machina, 'Expected utility' analysis without the independence axiom. Econometrica 50 (1982) 277–323.

[54] Mark J. Machina, Dynamic consistency and non-expected utility models of choice under uncertainty. Working paper, University of California, San Diego, Economics Department (October 1988).

[55] Mark J. Machina, Choice under uncertainty: Problems solved and unsolved. Econ. Perspectives 1(1) (1987a) 121–154.

[56] Mark J. Machina, Decision-making in the presence of risk. Science 236 (1987b) 537–543.

[57] Kenneth MacCrimmon, An experimental study of the decision making behavior of business executives. Unpublished doctoral dissertation, Graduate School of Management, University of California, Los Angeles (1965).

[58] Kenneth MacCrimmon and S. Larsson, Utility theory: axioms versus paradoxes. In: M. Allais and O. Hagen, eds., Expected Utility Hypotheses and the Allais Paradox (D. Reidel, Dordrecht, Holland, 1979).

[59] J. Quiggin, A theory of anticipated utility. J. of Econ. Behavior and Organization 3 (1982) 323–343.

[60] Zvi Safra, Uzi Segal and Avia Spivak, The Becker-deGroot-Marschak mechanism and nonexpected utility: A testable approach. Working paper, Segal, University of Toronto, Canada (May 1989).

[61] Rakesh K. Sarin, Analytical issues in decision methodology. In: Ira Horowitz, ed., Decision and Organization Theory (Kluwer-Nijhoff, Dordrecht, 1989).

[62] Uzi Segal, Nonlinear decision weights with the independence axiom. Working paper, Economics Department, UCLA, Los Angeles, CA 90024 (1984).

[63] Amos Tversky, Additivity, utility, and subjective probability. J. Math. Psych. 4 (1967) 175–201.

[64] Amos Tversky, Shmuel Sattath and Paul Slovic, Contingent weighting in judgment and choice. Psych. Rev. 95(3) (1988) 371–384.

[65] John von Neumann and Oskar Morgenstern, Theory of Games and Linear Programming, second ed. (John Wiley & Sons, New York, 1947).

[66] Detlof von Winterfeldt and Ward Edwards, Decision Analysis and Behavioral Research (Cambridge University Press, Cambridge, 1986).

[67] Martin Weber and Colin Camerer, Recent developments in modelling preferences under risk. OR Spektrum 9 (1987) 129–151.

[68] Menachem E. Yaari, The dual theory of choice under risk. Econometrica 55 (1987) 95–115.

TAB 11

Disappointment in Decision Making under Uncertainty
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Disappointment in Decision Making Under Uncertainty

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(Received August 1982; accepted February 1984)

Decision analysis requires that two equally desirable consequences should have the same utility and vice versa. Most analyses of financial decision making presume that two consequences with the same dollar outcome will be equally preferred. However, winning the top prize of $10,000 in a lottery may leave one much happier than receiving $10,000 as the lowest prize in a lottery. This paper explores the implications of disappointment, a psychological reaction caused by comparing the actual outcome of a lottery to one's prior expectations, for decision making under uncertainty. Explicit recognition that decision makers may be paying a premium to avoid potential disappointment provides an interpretation for some known behavioral paradoxes, and suggests that decision makers may be sensitive to the manner in which a lottery is resolved. The concept of disappointment is integrated into utility theory in a prescriptive model.

YOUR BOSS tells you that he is delighted with your performance over the past year and is giving you a $5,000 bonus. Are you pleased? If you were not expecting a bonus, you will be delighted. If you were expecting a $10,000 bonus, you will be disappointed. The satisfaction you feel with the bonus you are given will depend upon your prior expectations. The higher your expectations, the greater will be your disappointment. People who are particularly averse to disappointment may learn to adopt a pessimistic view about the future.

If you accept a 50-50 gamble between $0 and $2,000, there is a 50% chance that you will be disappointed when the lottery is resolved. You may prefer to swap the lottery ticket for a sure $950 not so much because of arguments about decreasing marginal value, but because doing so removes the possibility of disappointment. Of course, someone who feels that the "thrill of victory" is worth the possible "agony of defeat" may take the opposite choice.

Disappointment, then, is a psychological reaction to an outcome that does not match up to expectations. The greater the disparity, the greater the disappointment. We will use the word elation to describe the euphoria associated with an outcome that exceeds expectations. Decision makers who anticipate these feelings may take them into account when compar-

Subject classification: 94 reference effects, 852 disappointment as an attribute.

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ing uncertain alternatives. The purpose of this paper is (i) to model disappointment and to investigate the behavioral implications for a decision maker who is prepared to make economic tradeoffs to reduce disappointment, and (ii) to compare these implications with known behavioral violations of expected utility maximization when only the amounts of economic payoffs are taken into account.

This paper is a continuation of my research into the components of risk aversion. The economic explanation of risk aversion, that each additional dollar is worth slightly less due to satiation (decreasing marginal value), is surely a significant factor, but perhaps not the only one. Imagine two individuals with equal wealth and identical tastes for consumables. One is timid, nervous, and full of self-doubt; the other is outgoing, self-confident, and with a sense of purpose. We might suppose that the latter will be less risk averse than the former. Indeed, his relative risk attitude, in the sense of Dyer and Sarin [1982], may even be risk prone.

In earlier work (Bell [1982, 1983], also Loomes and Sugden [1982]), I have explored the implications of regret as a factor in risk attitude. Regret is a psychological reaction to making a wrong decision, where wrong is determined on the basis of actual outcomes rather than on the information available at the time of the decision. Just as disappointment is caused by comparing an outcome with prior expectations, so regret is caused by comparing an outcome with the payoff one could have had by making a different choice. For example, if you are given a 50-50 lottery between $0 and $10 and lose, you will suffer disappointment. Had you *selected* the same lottery over an alternative of $4 for sure and lost, you would have suffered both disappointment and regret. For one to suffer only regret and not disappointment, the outcome of a chosen lottery would have to be exactly equal to one's expectations, but less than one could have obtained (ex post) from an alternative lottery.

There are many other "reference effect" phenomena. A bonus of $5,000 may exceed one's expectations, but still lead to dissatisfaction if you learn that your colleague got a bonus of $10,000. Perhaps the most influential reference point is the status quo of the decision maker. It has been widely observed that a decision maker will make significant economic tradeoffs to remove the possibility of a net loss on a transaction.

Building a utility model that incorporates all of these effects may well be desirable not only to provide a better description of behavior, but also, to the extent that a decision maker is prepared to trade off dollars explicitly to gain a state of psychological satisfaction, for prescriptive purposes.

In this paper, to avoid unnecessary complication, we will consider only the effect of disappointment (and elation) on decision making under uncertainty. We will assume throughout that the decision maker has constant marginal value for money and never suffers from regret, from envy, or from a tendency to overweigh losses. The models that follow are perfectly adaptable to the case of nonconstant marginal value for money, by making an appropriate transformation of the attribute scale. A future paper will explore the implications of a model that incorporates decreasing marginal value and a variety of reference effects.

In Section 1 we examine systematic violations of the substitution principle of utility theory and show that a simple model incorporating disappointment offers an explanation for them. In Section 2 we present assumptions about the way disappointment and elation affect a decision maker's preference for outcomes and derive a preference model that will be used throughout the remainder of the paper. Section 3 shows how one might assess the components of the model in an interview with a decision maker. We show how a reasonable attitude with respect to disappointment can lead to risk prone behavior.

In Section 4 we consider how disappointment might lead to a preference by the decision maker for different methods of resolving a lottery. Perhaps the decision maker would prefer a 25% chance at $1,000 to be resolved by two sequential flips of a coin rather than by drawing a diamond from a deck of cards. In this section we show how violations of the substitution principle, in the context of a single dollar attribute, might be caused by such preferences. We also consider how news should be broken to people. An analysis of disappointment is compatible with the empirically learned behavior that bad news should be broken gently. Finally, we examine the implications of these matters for the relative desirability of different auction procedures.

1. A SIMPLE MODEL OF DISAPPOINTMENT

The substitution principle is key to the derivation of expected utility theory. Though some regard it as virtually a self-evident requirement for consistency, in much the way that the transitivity principle seems so compelling, others attack it as the weak link that allows behavioral contradictions in practice. The principle asserts (see Keeney [1982] for example) that any preference ordering of two alternatives with uncertain consequences will not be affected either by substituting for any one consequence an equally desirable consequence (which may itself be a lottery) or by deleting a consequence common to both alternatives. This principle is usually the one relied upon to demonstrate inconsistency in preference orderings.

We will use an example of Kahneman and Tversky [1979] to illustrate both the compelling nature of the substitution principle and why disappointment provides an explanation for some observed violations of it. (See also the discussion by Machina [1981 pp. 172–173].) Kahneman and

Tversky report that a prize of $3,000 for sure is preferred by a majority of respondents over an 80% chance at $4,000 (and a 20% chance at nothing), whereas a majority prefer a 20% chance at $4,000 over a 25% chance at $3,000. The following logic, relating to three decision situations illustrated in Figure 1, asserts that such behavior is undesirable.

Diagram *A* in Figure 1 shows the first of the two decision problems, diagram *B* shows the second. Diagram *C* illustrates a third situation in which a preliminary drawing determines whether there is anything to play for; with probability 0.25 the decision maker obtains the right to make a decision identical to that in diagram *A*.

The argument underlying the substitution principle is that it seems impossible to find an economic rationale for taking a different decision in diagram *C* from that taken in diagram *A*. Certainly the financial implications are identical in the two cases. Now we argue that the decision in diagram *B* should be the same as that in diagram *C*. Suppose we know

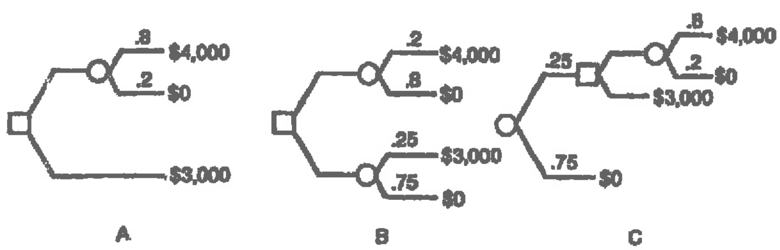

Figure 1. The substitution principle implies that either the upper branches or the lower branches should be selected at all decision forks.

that the decision maker will choose the $3,000 alternative in diagram *C*. Then there is no loss to the decision maker in announcing this choice at the outset. The only difference between situations *C* and *B* is whether the uncertainty is resolved in two stages or one. Since there are no economic implications to such a distinction, there should be no difference in the decision.

It is a common observation that people will agree with the step-by-step logic of the above argument, but nonetheless feel uncomfortable with the conclusion. The hypothesis of this paper is that psychological feelings of disappointment are ignored in the rational economic analysis, but play a role in the informal evaluation of alternatives by decision makers.

Consider the disappointment you would feel in situation *A* if you chose to gamble and got nothing. You would have had high expectations—an 80% chance of $4,000—suddenly dashed. If this prospect alarms you, the $3,000 prize for sure may look very attractive by comparison. In situation, *B*, however, I would not anticipate great disparity between the disappointment that I would feel at losing either of the two lotteries. In neither case do I have much chance of winning, so losing is almost to be expected, and the news of my loss reduces my expected asset value by only \$750 or \$800 compared to \$3200 in situation *A*.

This observation highlights where the logic behind the substitution principle breaks down. It is not a matter of indifference to me how a lottery is resolved. In situation *B* I may prefer to go with the \$4,000 gamble on the grounds that it has a higher expected value and the disappointment implications are similar for each alternative. However, in situation *C*, if the first stage is successful, my expectations rise dramatically and I become afraid of losing what I have gained.

A simple model will demonstrate that the above explanation is coherent. If someone owns an unresolved lottery having a probability p of yielding \$$x$ and a probability $(1-p)$ of yielding \$$y$ his psychological expectation may be supposed to be reflected by the quantity $px + (1-p)y$. (We will denote such a gamble by (x, p, y) where x is at least as preferred as y and p is the probability of winning.) If y occurs, the decision maker's disappointment might be in direct proportion to the difference between what he expected and what he got:

$$\text{Disappointment} = d(px + (1-p)y - y) = dp(x-y) \qquad (1)$$

where $d \geq 0$ is a constant reflecting the degree to which a unit of disappointment affects the decision maker. If x occurs, there will be a sense of elation which we may suppose is proportional to the difference between what the decision maker expected and what he got:

$$\text{Elation} = e(x - px - (1-p)y) = e(1-p)(x-y) \qquad (2)$$

where $e \geq 0$ is a constant reflecting the degree to which a unit of elation affects the decision maker. Equations 1 and 2 are identical except that elation and disappointment have been defined as positive quantities and the constants e and d, reflecting tradeoffs for the decision maker between dollars and psychological well-being, have been permitted to differ. To this point, we have implicitly assumed that the decision maker's multiattribute preference over dollars and disappointment (elation) is linear and additive:

$$\text{Total utility} = \text{economic payoff} + \text{psychological satisfaction}$$

where psychological satisfaction is positive for elation and negative for disappointment.

Two special cases are worth highlighting. If $p = 0$, i.e, there is no chance to win, then losing is to be expected and by (1) the level of disappointment is zero, appropriately enough. Similarly, if $p = 1$, the elation, by (2), is zero. If $d = e$, i.e., if disappointment and elation are equally compelling, then they cancel out when taking expectations:

$$p(\text{elation}) + (1 - p)(-\text{disappointment}) =$$
$$p[e(1 - p)(x - y)] + (1 - p)[-dp(x - y)] = 0.$$

Note that, although in this case disappointment and elation have no influence on *decisions*, they still do affect the desirability of individual outcomes.

But suppose one effect is more powerful than the other? If a decision maker suffers greatly when disappointed, but is relatively less influenced by elation ($e > d$), then the gamble (x, p, y) will have a certainty equivalent of

$$px + (1 - p)y + (e - d)p(1 - p)(x - y). \qquad (3)$$

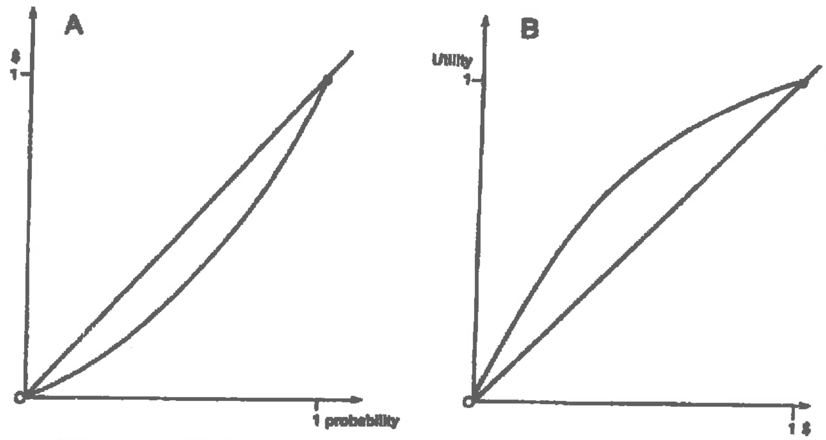

Figure 2. (A) Dollar certainty equivalents for (1, p, 0) lotteries. (B) Utility function for money implied by (A).

Figure 2(A) graphs the certainty equivalents of (1, p, 0) gambles as a function of p. Notice that even though we have assumed constant marginal value for money, a relative aversion to disappointment over elation will cause risk averse behavior by the decision maker. The certainty equivalents graphed in Figure 2(A) are entirely consistent with an explanation that says the decision maker has decreasing marginal value for dollars and is an expected utility maximizer using the utility function $u(x)$ over dollars defined implicitly by the equation

$$u(p + (e - d)p(1 - p)) = p$$

or, explicitly, if we let $k = d - e$, by

$$u(x) = (k - 1 + \sqrt{(1 - k)^2 + 4kx})/2k. \qquad (4)$$

This function is graphed in Figure 2(B). It has three properties often desirable in a utility function (Keeney and Raiffa [1976, p. 166]); it is increasing (if $e - d > -1$), it is concave (if $d > e$) and it exhibits decreasing risk aversion.

However, the decision maker does not obey the substitution principle using economic payoffs alone. Table I gives the certainty equivalents, using (3), for the four alternatives used in Figure 1. To reflect the empirically observed rank orders requires only that

$$3000 > 3200 + 640\,(e - d)$$

and

$$800 + 640\,(e - d) > 750 + 563\,(e - d)$$

or

$$-0.65 < e - d < -0.31.$$

Kahneman and Tversky have characterized a class of substitution

TABLE I

Certainty Equivalent Using the Simple Disappointment Model for Selected Lotteries

Lottery	Certainty Equivalent
(3000, 1, 0)	3000
(4000, 0.8, 0)	$3200 + 640\,(e - d)$
(3000, 0.25, 0)	$750 + 563\,(e - d)$
(4000, 0.2, 0)	$800 + 640\,(e - d)$

principle violations referred to as the *common ratio effect* as follows:

For $x > y > 0$ and $1 \geq p > 0$, $1 > q, r > 0$, if $(x, q, 0) \sim (y, p, 0)$ then $(x, qr, 0) > (y, pr, 0)$.

This behavioral rule is predicted by the simple formulation of this section if and only if $d > e$. For if $(x, q, 0) \sim (y, p, 0)$, then

$$qx + (e - d)q(1 - q)x = py + (e - d)p(1 - p)y. \qquad (5)$$

In order for $(x, qr, 0) > (y, pr, 0)$, we require

$$qrx + (e - d)qr(1 - qr)x > pry + (e - d)pr(1 - pr)y. \qquad (6)$$

Multiply (5) by r^2 and subtract from (6) to give the requirement that

$$qxr(1 - r)(1 + e - d) > pyr(1 - r)(1 + e - d)$$

which will be true if $qx > py$. From (5), this inequality will be true if $1 + (e - d)(1 - q) < 1 + (e - d)(1 - p)$ and, in particular, if $(e - d)(p - q) < 0$. Since $x > y$ and $e - d > -1$, we know that $(x, p, 0) > (y, p, 0)$ which implies $p > q$. Hence, the result is true as long as $e < d$.

Using Equation 3, we might interpret the quantity $(d - e)p(1 - p)\cdot(x - y)$ as a measure of the risk involved in the gamble. It is, after all, the reduction in the certainty equivalent caused by the presence of uncertainty. This measure is very similar to the variance measure of risk, common in financial applications, namely $\lambda p(1-p)(x-y)^2$ for some constant λ. Note that the term in $(x-y)$ is linear in (3), but squared in the variance formula. Stone [1973] gives a three parameter family that generalizes most of the well-studied risk measures. In the special case of (x, p, y), this family is $p|c-x|^{\alpha}\delta + (1-p)|c-y|^{\alpha}$ where c, α and δ are the three parameters, δ being 0 or 1. This family has the common property that all deviations from a reference point "c" are considered bad, contributing positively to the risk measure. By contrast, the measure $(d-e)p(1-p)(x-y)$ was derived in a way that permits "good deviations" and is not a member of Stone's family.

The variance is a natural measure of risk if one is trying to reflect the effects of decreasing marginal value, because of the approximate relationship (Pratt [1964]):

$$Eu(x) \simeq u(0) + E(x)u'(0) + \tfrac{1}{2}E(x - E(x))^2u''(0)$$

where primes represent differentiation. Our simplifying assumption that the marginal value of money is constant removes this natural advantage of the variance. Although much empirical work has been done to identify simple statistics that explain hypothetical choices made by businessmen (Wehrung et al. [1978] and MacCrimmon and Wehrung [1983]) no analysis has been done, to my knowledge, on (3), perhaps because it has no obvious analog for continuous distributions. Although this paper studies only two outcome gambles, it is possible to deduce a generalization of (3) from the assumptions in Section 2, namely, that the certainty equivalent of a distribution with mean μ and variance σ^2 is $\mu - k\sigma$ where k depends not only on the decision maker, but also on the particular characteristics of the distribution. Two distributions that are related by a simple change of scale (see Assumptions 6 and 7) will have the same k value, so that, for example, all normal distributions will have the same k value. Note that (x, p, y) and (x', p, y') are related by a simple scale change, but that (x, p, y) and (x, q, y) are not. Formula (3) fits the format $\mu - k\sigma$ when $k = (d-e)\sqrt{p(1-p)}$.

The Joy of Winning

The simple representations (1), (2), and (3) were introduced to illustrate that the idea of disappointment can be modeled and applied systematically to decision situations. Since the disappointment a decision maker feels at an outcome is likely to be sensitive to context, no simple model will be an accurate representation in all circumstances. The formation of expectations may vary from person to person. While a mathematician may expect the probabilistic average, an optimist may expect more, a pessimist less. In multiple outcome gambles one's expectations may be the mode. A decision analyst may feel it appropriate to expect the certainty equivalent, an exercise (fortunately convergent) in cyclical reasoning, which may explain why decision analysts take so long to make decisions.

The gambles in Figure 3 may demonstrate that the simple model is not sufficiently rich. All four lotteries have an expected value of $1,000 so that, according to (1), the disappointment on receiving $0 should be the same in all four of them. Many people might feel that the disappointment would be greatest in the fourth case where the chance of losing was so remote, despite the fact that less was lost. The important question for our model is whether or not the level of disappointment is the same in all four cases.

Consider a 50-50 lottery between $0 and $2,000 and also a lottery with a normal distribution having mean $1,000 and standard deviation of $10.

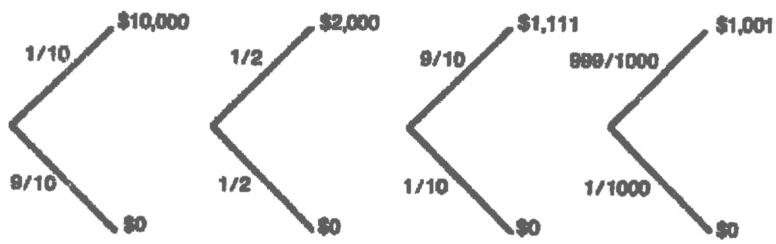

Figure 3. Test of simple disappointment model: Is the level of disappointment the same at all $0 outcomes?

Compare your reaction to receiving $0 in each case. The disappointment may be greater in the second case than in the first. In the first lottery, the zero outcome had a 50% chance of occurring and bordered on being "expected." In the second lottery, an outcome as low as $0 was virtually impossible and a feeling of great dismay would be understandable. Even if the example is not so extreme, say a standard deviation of $1,000, the outcome $0 is still worse than about 84% of the distribution. It may be disappointing to do so badly.

This discussion suggests that disappointment may be related not only to a level of prior expectations, but also in a direct way to the likelihood with which the outcome occurred. Even if you were not expecting to win much, it is discouraging to lose, especially when the odds were against it. On the other hand, it is plain fun to win things. It is exciting to win at party games even if the skill required is none and the prize paltry. There is a thrill to be had from being dealt an unusually good bridge hand quite apart from any significance it may have to the course of the game. Figure

4 shows gambles that have been constructed with the same top prize of \$10,000 and expected values equal to \$9,000. According to (2), the elation should be the same on winning each of them. Many people might feel that the elation goes up with the odds against winning. (When looking at the lotteries, do not forget to adjust your expectations to allow for owning the lottery, *before* considering the elation on winning.)

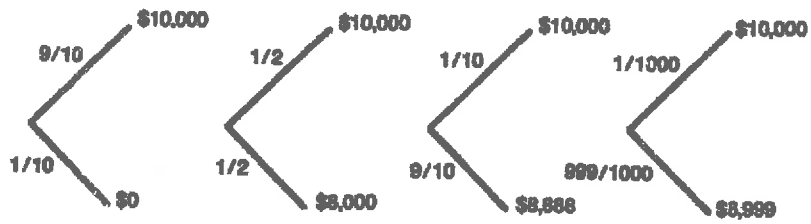

Figure 4. Test of simple elation model: Is the level of elation the same at all \$10,000 outcomes?

2. BASIC ASSUMPTIONS

Despite the above complications about how expectations are formed and whether elation and disappointment depend directly upon the probabilities involved, there is a way to model these concepts that avoids the necessity of being explicit on these matters and permits a straightforward assessment task on the part of the decision maker. The assumptions needed for this model are covered in this section.

Let $L_0(x, p, y)$ represent the state of owning an unresolved lottery (x, p, y). Let $L_1(x, p, y)$ represent the outcome in which $L_0(x, p, y)$ results in x. Let $L_2(x, p, y)$ represent the outcome in which $L_0(x, p, y)$ results in y. (Recall that we always assume $x \geq y$.)

ASSUMPTION 1 (Simple Orderings). *For* $i = 0, 1, 2$, *the decision maker can make preference comparisons of the form*

$$L_i(x_1, p_1, y_1) >, \sim \quad or \quad < L_i(x_2, p_2, y_2).$$

These orderings are transitive.

Of course, a fair degree of introspection is required to say whether winning the top prize from the lottery (100, 1/10, 0) is better or worse than winning the top prize from (110, 1/2, 0). Although this talent is required in principle, it will not be called upon in assessment procedures.

ASSUMPTION 2 (Sure-thing Indifference). *For* $i = 0$ and 1, $L_i(x, p, x) \sim L_i(x, 1, y)$ *and for* $i = 0$ *and* 2, $L_i(y, p, y) \sim L_i(x, 0, y)$.

The decision maker is presumed not be affected by the presence of impossible consequences.

ASSUMPTION 3 (Monotonicity in the Probability of Winning). *If* $p > q$, *then*

$$L_0(x, p, y) > L_0(x, q, y)$$

and

$$L_i(x, p, y) < L_i(x, q, y) \quad \text{for} \quad i = 1, 2.$$

Note that these two conditions are compatible because $L_0(x, p, y)$ is more likely than $L_1(x, q, y)$. It is conceivable that someone may actually prefer (1, ⅕, 0) over (1, ¼, 0) because of the greater excitement on winning, but we are content to exclude such people from our theory.

ASSUMPTION 4 (Monotonicity in Value of Prizes). *If* $x' > x$, *then*

$$L_i(x', p, y) > L_i(x, p, y) \quad \text{for} \quad i = 0, 1$$

and

$$L_2(x', p, y) < L_2(x, p, y).$$

If $y < y'$, *then*

$$L_i(x, p, y) < L_i(x, p, y') \quad \text{for} \quad i = 0, 2$$

and

$$L_1(x, p, y) > L_1(x, p, y').$$

Assumption 4 implies that $(x, p, x) > (x, p, y) > (y, p, y)$ so that it is reasonable to suppose that there exists a value c_0 with $(c_0, p, c_0) \sim (x, p, y)$. As long as the effects of disappointment/elation are not infinitely great, it is also appropriate to posit certainty equivalents for the winning and losing outcomes.

ASSUMPTION 5 (Solvability). *For all* x, p *and* y *and* $i = 0, 1, 2$ *there exists a* c_i *depending on* x, p *and* y *such that*

$$L_i(c_i, p, c_i) \sim L_i(x, p, y).$$

The functions $c_i(x, p, y)$ are the cash certainty equivalents of the situations $L_i(x, p, y)$. The quantity $c_0(x, p, y)$ is simply the traditional concept of a certainty equivalent.

Although we do not seek to model prior expectations explicitly, it seems reasonable to suppose that our prior expectations for the gamble $(x + k, p, y + k)$ will be exactly an amount k higher than those for (x, p, y). Similarly, my expectations for $(2x, p, 2y)$ would be twice those for (x, p, y). (Our assumption of a constant marginal value for money is important here. More generally, we would say that the *value* of the prizes must double in order for expectations to double.) In short, while not explicitly modeling the formulation of expectations, we will assume, at least implicitly, that they are linear in the payoffs.

ASSUMPTION 6 (Constant Marginal Value for Payoffs). *For all* x, p, y, a

and b $(b > 0)$, *and* $i = 1, 2$

$$c_i(a + bx, p, a + by) = a + b \cdot c_i(x, p, y).$$

This assumption is based on a presumption that expectations are linear in the payoffs and that, for a given p, disappointment and elation are proportional to the difference between outcome and expectations.

ASSUMPTION 7 (Risk Neutrality in the Absence of Disappointment). *For all* x, p *and* y,

$$c_0(x, p, y) = pc_1(x, p, y) + (1 - p)c_2(x, p, y).$$

The underlying presumption is that the decision maker would be risk neutral if it were not for the effects of disappointment and elation. Once again, this presumption derives from our assumption of constant marginal value for money. Since $c_1(x, p, y)$ and $c_2(x, p, y)$ are the cash equivalents for the outcomes, the assumption follows.

THEOREM 1. *Assumptions* 1, 2, 5, 6 *and* 7 *imply that for* $i = 0, 1,$ *and* 2, *the situations* $L_i(x, p, y)$ *have certainty equivalents of* $y + (x - y)\pi_i(p)$ *for some functions* π_i.

Proof. Assumptions 1 and 5 allow us to establish the functions $c_i(x, p, y)$ used in Assumptions 6 and 7. Assumption 6 implies that $c_i(x, p, y) = y + c_i(x - y, p, 0) = y + (x - y)c_i(1, p, 0)$ for $i = 1, 2$. Assumption 2 is merely Assumption 6 in the special case $b = 0$, or $x = y$. Assumption 7 shows that Assumption 6 also applies in the case $i = 0$. Let $\pi_i(p) = c_i(1, p, 0)$.

For the remainder of the paper we will adopt a more mnemonic notation in place of the π_i's. We will replace π_0 simply by π. This choice highlights the similarity between our result and that used by Kahneman and Tversky [p. 276, Equation 2] in their prospect theory. Had we chosen to retain the possibility of a nonconstant marginal value for money by use of a nonlinear value function $v(x)$, our assumptions would have led to the model

$$v(y) + \pi(p)(v(x) - v(y)). \qquad (7)$$

One major distinction between our model and that of prospect theory is that we make no distinction between positive and negative payoffs; that is, we do not take account of the status quo as a reference point. For example, prospect theory evaluates the gamble (x, p, y) if $0 > x > y$ as $v(x) + \pi(p)[v(y) - v(x)]$ whereas the disappointment formula continues to evaluate this gamble as in (7). It is worth repeating that reference points such as status quo, a foregone outcome (regret), and an assumption of nonconstant marginal value are excluded from our current analysis, only because their presence would complicate both the analysis and our understanding of the effect disappointment has on decision making, not because these factors are unimportant.

Despite these differences with prospect theory, the notation $\pi(p)$ is worth adopting because of the common interpretation of π as a behavioral subjective probability, since the decision maker acts in accordance with simple expected values once this transformation of probability is made. (This terminology for π applied to the traditional utility model would call $u(x)$ a behavioral subjective value.)

Instead of using $y + (x - y)\pi_1(p)$ as the certainty equivalent of the winning situation, we will use $x + (x - y)w(p)$. The function $w(p)$ may be thought of more directly as the value of the elation that comes with winning in the lottery $(1, p, 0)$. Similarly, in place of $y + (x - y)\pi_2(p)$ as the certainty equivalent of the losing outcome we will use $y - (x - y)$ $l(1 - p)$. The function $l(p)$ may be thought of as the (positively valued) psychological cost of losing in the gamble $(1, 1 - p, 0)$. With this notation, $w(p)$ and $l(p)$ are each related to an outcome that occurs with probability p.

To summarize, we have

$$c_0(x, p, y) = y + (x - y)\pi(p)$$

$$c_1(x, p, y) = x + (x - y)w(p) \tag{8}$$

and

$$c_2(x, p, y) = y - (x - y)l(1 - p).$$

The following properties of the functions π, w and l are deducible from the assumptions:

(i) $\pi(0) = w(1) = l(1) = 1 - \pi(1) = 0$ (Assumption 2) (9)

(ii) π is increasing, w and l are decreasing functions of p (Assumption 3) (10)

(iii) $\pi(p) = p + pw(p) - (1 - p)l(1 - p)$ (Assumption 7). (11)

This model is very flexible for our purposes. The simple model used in Section 1 is a special case, where

$$w(p) = (1 - p)e, \qquad l(p) = (1 - p)d,$$
$$\pi(p) = p[1 + (e - d)(1 - p)]. \tag{12}$$

Even if we presume that the certainty equivalent of the gamble is also the prior expectation, we obtain a model in the same class. That is, suppose we have

$$c_2(x, p, y) = y + [y - c_0(x, p, y)]d$$

and

$$c_1(x, p, y) = x + [x - c_0(x, p, y)]e$$

as parallels to (1) and (2), then using Assumption 7 and solving these questions for c_0, we may deduce that

$$c_0(x, p, y) = y + (x - y)\pi(p)$$

where
$$\pi(p) = p(1 + e)/[1 + ep + d(1 - p)].$$

We also can show that $w(p) = (1 - \pi(p))e$ and $l(p) = d\pi(1 - p)$.

3. ASSESSMENT PROCEDURES

If we are interested only in the effects of disappointment on decision making, then only the function π need be assessed, which may be done by the obvious mechanism of asking directly for certainty equivalents for the gambles $(1, p, 0)$. However, it would be important, in any prescriptive analysis that incorporates disappointment, for the assessment procedure to require explicit tradeoffs between psychology and economy. Assessment of the functions w and l requires the decision maker to compare outcomes (and the psychological consequences that go with them) instead of alternatives. Such procedures are open to question on the grounds that decision maker responses are not testable in any satisfactory manner and because it is easy to imagine situations where discrepancies occur. (For example, you may choose to work late rather than go home, but you wish that the option of working late were unavailable. This example was inspired by Schelling [1983].)

The following assessment procedure is presented to show the kinds of inputs that are required. Such an assessment procedure need not be routine.

A straightforward assessment procedure for $w(p)$ begins by asking the decision maker to identify an amount $\$k_1(p)$ such that winning \$1 from $(1, p, 0)$ is just as satisfying as winning \$1 from $(1, ½, k_1(p))$. (Note that the upper and lower prizes are flexible, they could be \$1000 and 0, for example). From this answer, we obtain indifferences of the form

$$c_1(1, ½, k_1(p)) = c_1(1, p, 0). \tag{13}$$

Since (13) is equivalent to the equation

$$1 + (1 - k_1(p))w½ = 1 + w(p) \tag{14}$$

we have, by varying p, assessed $w(p)$ up to the constant w (½). Similarly, if $k_2(p)$ is the dollar amount such that

$$c_2(k_2(p), (½), 0) = c_2(1, p, 0),$$

then

$$-k_2(p)l(½) = -l(1 - p)$$

or
$$l(p) = l(½)k_2(1 - p). \tag{15}$$

Note that so far we have not asked any questions that explicitly trade off psychological effects against dollars. The essence of these tradeoffs is contained in the constants $w(½)$ and $l(½)$.

Though these parameters could be obtained by direct questioning, it seems reasonable to think of recovering them implicitly via direct assessment of $c_0(1, p, 0)$. Using Equations 11, 14, and 15, we may deduce a relation between π, k_1 and k_2:

$$\pi(p) = p[1 + (1 - k_1)w(½)] - (1 - p)l(½)k_2.$$

Hence, any two direct assessments $\pi(p_1)$, $\pi(p_2)$ will serve to define $w(½)$ and $l(½)$.

Risk Prone Behavior

Studies of utility commonly make hypotheses about properties of the utility function that should hold for "most people." These studies generally assume that people are risk averse in monetary gambles and for the extent of their risk aversion (Pratt) to decrease as they become wealthier (Raiffa [1968, p. 91]).

Such generalizations in the case of preferences for disappointment are less well-founded, but the following one, which may be reasonable, not only suggests that risk aversion may increase with p, but also permits the possibility of risk proneness for small values of p. The assumption is based on the generic examples used in Figures 3 and 4. The gamble $(1/p, p, 0)$ has an expected value of 1 for all p. The assumption is that the disappointment, on receiving 0 from this gamble, increases with p. The gamble $(1, p, -p/(1-p))$ has an expected value of 0 for all p. The assumption is that the elation, on receiving 1 from this gamble, decreases with p.

ASSUMPTION 8. *For* $1 > p > q > 0$,

$$c_2(1/p, p, 0) < c_2(1/q, q, 0)$$

and $$c_1(1, p, -p/(1-p)) < c_1(1, q, -q/(1-q)).$$

THEOREM 2. *Assumption 8 implies that there exists a probability* p^*, *which might be zero or one, with the property that the decision maker is risk averse whenever* $1 > p > p^*$, *risk neutral for* $p = p^*$, *and risk prone for* $0 < p < p^*$.

Proof. Assumption 8 can be rewritten as

$$-l(1-p)/p < -l(1-q)/q$$

and $$1 + w(p)/(1-p) < 1 + w(q)/(1-q)$$

so that $w(p)/(1-p)$ and $l(p)/(1-p)$ are decreasing functions of p. The decision maker is risk averse so long as $\pi(p) < p$, which is true if $pw(p)$

$< (1-p)l(1-p)$ or $w(p)/(1-p) < l(1-p)/p$. We know that $w(p)/(1-p) - l(1-p)/p$ is decreasing in p so that either $\pi(p) - p$ has a constant sign for all p or there exists a critical value p^* as required.

If we assume for a moment that w and l are differentiable functions, we may establish a necessary and sufficient condition for $p^* > 0$, that is, that the decision maker is ever risk prone. For small p, $\pi(p) - p = pw(p) - (1-p)l(1-p) = pw(0) - (1-p)l(1) + pl'(1)$. Since $l(1) = 0$, the condition to be satisfied for risk proneness is $w(0) + l'(1) > 0$. In the simple formulation $w(p) = (1-p)e$, $l(p) = (1-p)d$ we have $w(0) = e$, $l'(1) = -d$, so that this inequality becomes $e > d$. In this case, the decision maker is either always risk prone or always risk averse. Figure 2(A) was drawn assuming $e < d$.

Supporting the Underdog

Frequently, one is watching a sports event involving two teams, without any intrinsic reason for supporting either. While I know of no formal study to support these conclusions, two observations seem to be widely acknowledged. The first is that spectating is more fun if you choose a team to support and the second is that uncommitted people tend to support the underdog, the team thought less likely to win. Why is this? Certainly, I prefer the team I am supporting to have a high probabilty of winning (attendance increases dramatically when the home team looks likely to win), yet when faced with a free choice I choose to support the team with the lesser chance.

Suppose the spectator believes that the underdog has a probability p of winning, where $p < ½$. Since there is no intrinsic reason for supporting either team, the only payoff to the spectator is psychological. If the team he supports wins he is elated; if it loses he is disappointed. The expected psychological benefits from supporting the favorite and underdog are modeled as if something was at stake, but no economic value is assigned:

$$\text{Favorite:} \quad (1-p)w(1-p) - pl(p)$$

$$\text{Underdog:} \quad pw(p) - (1-p)l(1-p).$$

Supporting the underdog makes sense if $p[w(p) + l(p)] > (1-p)[w(1-p) + l(1-p)]$ or

$$w(p)/(1-p) + l(p)/(1-p) > w(1-p)/p + l(1-p)/p.$$

Since $w(p)/(1-p)$ and $l(p)/(1-p)$ are decreasing functions, this equation will hold if and only if $p < ½$.

Supporting *somebody* is a good idea of $pw(p) - (1-p)l(1-p) > 0$ or $\pi(p) - p > 0$. A spectator will get the most enjoyment out of watching a game where the underdog has a probability of winning that maximizes the quantity $\pi(p) - p$. If $\pi(½) < ½$, this analysis suggests that in a close contest it might be wiser not to support anyone, and that this position is always true for those that dislike any form of gambling.

Political contests are another example where there is evidence of support from uncommitted people to the underdog. According to our model, if an undecided voter goes to vote, he or she should vote for the underdog.

Risk prone behavior and supporting the underdog are, by our model, one and the same phenomenon. Supporting a cause with little chance of success offers only the possibility of elation and eliminates the possibility of sizeable disappointment. It is a case of minimizing the downside risk.

An Example: Reacting to the Odds

As a simple example of nonlinear w and l functions, let us suppose that instead of being proportional to the *probability* of winning (losing), elation (disappointment) is proportional to the *odds* against the event occurring. Winning one dollar at odds of 10 to 1 is presumed to be twice

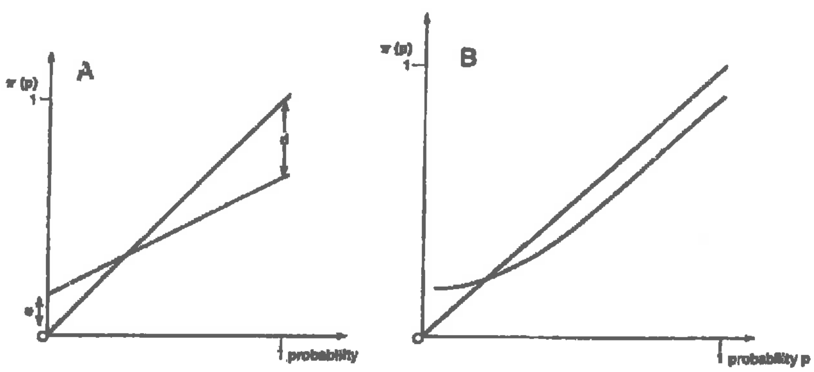

Figure 5. (A) A function $\pi(p)$ deduced from a disappointment model. (B) The function $\pi(p)$ derived from experimental data by Kahneman and Tversky.

as elating as winning one dollar at odds of 5 to 1. We assume $w(p) = (1-p)e/p$ and $l(p) = (1-p)d/p$ where e and d are constants, not necessarily equal to those in (3). There is some question as to the appropriateness of this illustration as $p \rightarrow 0$ since it implies infinite disappointment for rare bad events (nuclear power?), so we must use caution on results for extreme probabilities. However, with this model we have

$$c_2(1/p, p, 0) = -l(1-p)/p = -d/(1-p)$$

which decreases with p, agreeing with Assumption 8. Also, $c_1(1, p, -p/(1-p)) = 1 + e/p$ which decreases with p, also in accordance with

Assumption 8. The function π has an especially simple form, namely

$$\pi(p) = p + pw(p) - (1-p)l(1-p) = p + (1-p)e - pd$$

or

$$\pi(p) = e + (1 - e - d)p \qquad 0 < p < 1. \tag{16}$$

To satisfy Assumption 3, we require $e + d < 1$. Figure 5 shows a graph of this straight line function next to the π function of Kahneman and Tversky (their Figure 4). Both graphs are discontinuous at 0 and 1.

This odds model induces the common ratio effect in behavior discussed in Section 1. It also causes risk prone behavior for small p as long as $e > 0$. The underdog will always be supported in preference to the favorite.

The empirical results of Preston and Baratta [1948] suggest that the breakeven point p^* where $\pi(p^*) = p^*$ occurs around $p^* = ¼$. This result implies that $d = 3e$. On an absolute level, their results suggest a value for e of around 0.1.

4. SENSITIVITY TO METHOD OF UNCERTAINTY RESOLUTION

In Section 2 we developed a model that captures the idea that people's reaction to decision outcomes is a function not only of the absolute value of their payoff, but also to the change in their expectations and to the likelihood of such outcomes. So far we have looked at the implications of disappointment for decision making in standard situations including violations of the substitution principle and risk proneness.

Now we extend our analysis to the implications of disappointment for a class of decision situations that has received little empirical study and almost no normative analysis, the possible preference of the decision maker for different methods for resolving what are otherwise identical situations.

The logical argument underlying the substitution principle *fails* if we agree that the method of resolution may matter. While the economic implications may not change, the psychological implications might.

For example, the Irish Sweepstakes is run as a two-stage lottery. In the first stage, 20 or so tickets are drawn at random to match one for one the horses running in the Irish Derby. At the second stage, you win the lottery if you draw a horse at the first stage and if your horse wins the race. Since the prize money is significant, the disappointment from winning a horse, but not the race, would be substantial. The disappointment would be greatest had you drawn the favorite. Many first stage winners are offered sizeable amounts for their tickets. The threat of losing may make such an offer very attractive. (There are competing arguments however. The severe regret that would accompany having sold what proved to be the winning ticket might be intolerable. Also, as one such first stage winner put it, "My luck has been with me so far, why should I sell now?" The analysis here confines itself to the effects of disappointment.)

The general question is, when faced with a $(1, pq, 0)$ lottery, would you rather have it resolved in two stages, the first being $(1, q, 0), p, 0)$ and the second (if necessary) being $(1, q, 0)$, rather than all at once? Even though a substitution principle will not hold when disappointment is an issue if preference is modelled on assets alone (see Figure 1), a substitution principle *should* hold if assets and disappointment are assumed to be a complete description of the decision maker's concerns.

ASSUMPTION 9 (Backward Substitution). *If c_0 is the certainty equivalent of a lottery (x, p, y), then in a two stage lottery in which (x, p, y) is a possible prize of the first stage, a prize of c_0 for sure may be substituted for (x, p, y) without affecting the certainty equivalent of the entire two-stage gamble. In particular, the decision maker is just as satisfied to win c_0 as to win (x, p, y).*

THEOREM 3. *The certainty equivalent of a two-stage gamble* $((1, q, 0), p, 0)$ *is* $\pi(p)\pi(q)$.

Proof. By Assumption 9, we are indifferent between $((1, q, 0), p, 0)$ and $(\pi(q), p, 0)$. But this is equivalent to a sure-thing of $\pi(p)\pi(q)$.

In general, we have

$$\begin{aligned} c_0((x, q, y), p, (w, r, z)) &= c_0(y + (x - y)\pi(q), p, z + (w - z)\pi(r)) \\ &= z + (w - z)\pi(r) \\ &\quad + \pi(p)[y - z + (x - y)\pi(q) - (w - z)\pi(r)] \\ &= z + (y - z)\pi(p) + (w - z)\pi(r) \\ &\quad + (x - y)\pi(p)\pi(q) - (w - z)\pi(r)\pi(p). \qquad (17) \end{aligned}$$

THEOREM 4. *The certainty equivalent of two independent gambles to be resolved sequentially is equal to the sum of their individual certainty equivalents. In particular, the decision maker is indifferent to the order of their resolution.*

Proof. Suppose the lotteries are (x, p, y) and (w, q, z). One possible order of resolution is

$$((x + w, p, y + w), q, (x + z, p, y + z)),$$

the other is

$$((w + x, q, z + x), p, (w + y, q, z + y)).$$

Using (17), we see that the first of these lotteries has a certainty equivalent of

$$y + z + (w - z)\pi(q) + (x - y)\pi(p) + (x - y)\pi(p)\pi(q) - (x - y)\pi(p)\pi(q)$$

or

$$y + z + (w - z)\pi(q) + (x - y)\pi(p).$$

This quantity equals the sum of the separate certainty equivalents. Hence, the order of resolution is of no consequence.

Theorem 3 is counter-intuitive for me since it implies, for example, that I should be indifferent between ((1000, 1/80, 0), 4/5, 0) and ((1000, 4/5, 0), 1/80, 0). Yet, I much prefer the first lottery on the grounds that the only way to be seriously disappointed is if I take the second lottery and then win the first stage but lose the second. The theorem prompts me to notice that this dire consequence has only 1 chance in 400 of occurring and that other endpoints should carry more weight in my decision.

Backward induction does not tell us how best to resolve a given lottery. Which is better, (1, 1/18, 0), ((1, 1/6, 0), 1/3, 0) or ((1, 1/9, 0), 1/2, 0)? If we assume that the excitement of winning intensifies rapidly as the odds against winning increases, then we might conclude that the most preferred resolution procedure is that in which the odds against winning on a single resolution is greatest. Note that this procedure does not contradict Assumption 3 because we are comparing only lotteries with equal overall chance of winning. Hence, (1, 1/18, 0) is the best choice because our elation is $e(1/18)$ if we win. The lottery ((1, 1/6, 0), 1/3, 0) is least preferred because neither stage is particularly exciting, relative to $e(1/18)$ and $e(1/9)$.

Assumption 10 (Maximize the Maximum Elation). *If $p_1q_1 = p_2q_2$ and $p_1 < p_2 < q_2$, then*

$$((1, p_1, 0), q_1, 0) > ((1, p_2, 0), q_2, 0).$$

The odds model (16) satisfies this assumption. For

$$(e + (1 - e - d)p_1)(e + (1 - e - d)q_1) > (e + (1 - e - d)p_2)(e + (1 - e - d)q_2)$$

is equivalent to $p_1 + q_1 > p_2 + q_2$ when $p_1q_1 = p_2q_2$ and $e + d < 1$. Also, if $p_1q_1 = p_2q_2$, then $(p_1 + q_1)^2 > (p_2 + q_2)^2$ if and only if $(p_1 - q_1)^2 > (p_2 - q_2)^2$ which will hold when $p_1 < p_2 < q_2$.

The assumption implies, for example, that contestants in the Irish Sweepstakes Lottery should prefer fewer horses to be running since the thrill of drawing a horse on the first round will more than offset the increase in disappointment on the second round should that horse not win.

Assumption 10 has an interesting association with the common ratio effect.

THEOREM 5. *The common ratio effect and Assumption* 10 *have equivalent implications for the function* π.

Proof. The common ratio effect, as noted earlier, says that if $(x, p, 0) \sim (y, q, 0)$ where $x > y$, then if $0 < r < 1$, we have $(x, pr, 0) > (y, qr, 0)$. That is, if $x\pi(p) = y\pi(q)$, then $x\pi(pr) > y\pi(qr)$, from which we deduce that $\pi(pr)\pi(q) > \pi(qr)\pi(p)$ when $p < q$. Assumption 10 is equivalent, by Theorem 3, to $\pi(p_1)\pi(q_1) > \pi(p_2)\pi(q_2)$ when $p_1q_1 = p_2q_2$ and $|p_1 - q_1| > |p_2 - q_2| > 0$. Set $p_1 = pr$, $q_1 = q$, $p_2 = qr$ and $q_2 = p$. If $p < q$, then $|pr - q| > |qr - p|$. Hence, the common ratio effect and assumption 10 have equivalent implications for the shape of the function π.

The common ratio effect answers the original question posed in this section: since $\pi(pq) > \pi(p)\pi(q)$ the decision maker prefers the uncertainty resolved all at once rather than in stages. Note that Assumption 10 and the common ratio effect apply to only one form of two-stage gambles involving two outcomes. Quite different algebra is required to study gambles such as $(1, p, (1, q, 0))$.

Ellsberg's Paradox

Two urns filled with colored balls are before you. One has equal numbers of red and black balls, the other contains an unknown proportion of red and black balls. You choose an urn and a color. Then you draw a ball from your chosen urn. If the ball is the color you selected, you win a prize. Ellsberg [1961], who constructed this scenario, noted that people strongly preferred to draw from the first urn despite the fact that an economic analysis suggests no reason to prefer one over the other.

Suppose, should you choose the second urn, that after you have chosen a color but before you have picked a ball, you are told the true proportion of red and black balls. For simplicity, let us suppose that the probability of drawing a red ball is known in advance to be either r (which is $>½$) or $1 - r$, with equal likelihood.

Your choice, then, is between $(1, ½, 0)$ or $((1, r, 0), ½, (1, 1 - r, 0))$. The first urn will be preferred if

$$\pi(½) > \pi(1 - r) + \pi(½)(\pi(r) - \pi(1 - r)). \qquad (18)$$

This inequality is approximately equivalent to

$$0 > (2r - 1)(1 - 2\pi(½))\pi'(½)$$

as long as r is sufficiently close to ½ where π' is the derivative of π. In this case, the first urn will be preferred as long as $\pi(½) < ½$, that is, if

$w(\frac{1}{2}) < l(\frac{1}{2})$, a condition we expect if disappointment has a greater effect than elation.

The linear model (12) (or (3)) satisfies (18) exactly. The odds model (16) satisfies it exactly if $e < d$. Presentations of Ellsberg type choices usually do not discuss the precise mechanics of the resolution, and we do not know what participants assume about these mechanics. Though the interpretation given here may not be common, it serves to underscore the fact that the manner of resolution of lotteries, which has always been assumed to be irrelevant (since they are usually hypothetical anyway), is a potentially important element of the problem description.

Breaking Good/Bad News

So far our discussion has studied two-stage lotteries whose method of resolution is determined before the resolution takes place. The opposite order is possible if someone other than the decision maker already knows the outcome and can select the method of resolution in the decision maker's best interest.

For example, suppose that you have the job of telling someone that a close friend/relative has been killed in a car crash. How do you go about it? A normal reaction is to break the news in stages. ("I have some bad news. There's been a terrible accident. I'm afraid . . . . ") The interpretation is that you are gradually lowering the recipient's expectations so that the truth does not come all at once. Your mechanic knows this method too. ("It was a bigger job than we thought. We had to rebuild the engine. And replace the chassis . . . . ")

Breaking good news seems rather different. Suppose that you noticed your brother had inadvertently neglected to check the status of his Irish Sweepstakes ticket, and you found that not only had he drawn a horse but that the horse had won, how would you tell him the news?

It is arguable whether the method of breaking news is chosen more for the benefit of the teller than the tellee, but an analysis of the disappointment implications suggests that it will usually, but not always, be the case that bad news should be broken gently and good news all at once. Before our analysis begins one more assumption is required. So far, we have no explicit model of the net psychological effect of two sequential resolutions. How do you feel after winning one bet, but losing another? The following assumption was implicit in Assumption 9. It says that psychological effects accumulate additively.

ASSUMPTION 11 (Additive Psychological Effects). *If the decision maker is indifferent between* $\pi(q) + c$ *and winning* $\pi(q)$ *in the lottery* $(\pi(q), p, y)$*, then the decision maker is also indifferent between winning the lottery* $(1 + c, q, c)$ *and getting the prize of* 1 *in the lottery* $((1, q, 0), p, y)$*. A similar condition is assumed when the second stage lottery is lost, and when the first stage lottery is lost.*

We know that $c_1(\pi(q), p, y) = \pi(q) + (\pi(q) - y)w(p)$. Also, $c_1(1 + c, q, c) = 1 + c + w(q)$. Hence, winning a prize of 1 from $((1, q, 0), p, y)$ is, by Assumptioin 11, worth $1 + (\pi(q) - y)w(p) + w(q)$. The second two terms are just the elation values at each round.

Now consider the problem of breaking good news. In Figure 6(A), suppose that we know the lottery resulted in 1. Should we tell the owner of the gamble the net result giving him an elation value of $w(pq)$, or should we tell him step by step, just as the gamble was resolved, giving him an elation of $\pi(q)w(p) + w(q)$?

Good news should be broken abruptly if and only if

$$w(pq) \geq \pi(q)w(p) + w(q). \qquad (19)$$

Examination of Figure 6(B) shows that bad news should be broken gently

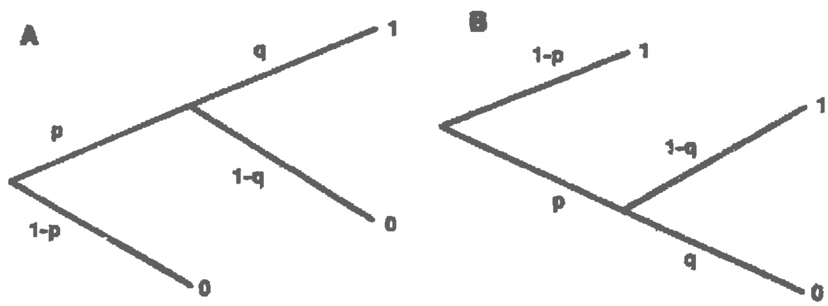

Figure 6. (A) One way to break good news. (B) One way to break bad news.

if and only if

$$l(pq) \geq (1 - \pi(1 - q))l(p) + l(q). \qquad (20)$$

For a general function f, the constraint $f(pq) \geq qf(p) + f(q)$ is satisfied if and only if $f(x)/(1 - x)$ is decreasing in x. Assumption 8 tells us that $w(x)/(1 - x)$ and $l(x)/(1 - x)$ are decreasing in x so that (19) and (20) would hold if $\pi(x) = x$. Inequality (19) will hold if $\pi(q) \leq q$, and inequality (20) will hold if $\pi(1 - q) \geq 1 - q$. Both of these conditions will be valid if q is sufficiently large, in particular, if it exceeds $1 - p^*$ (see Theorem 2). However, it only takes $w(p)$ and $l(p)$ to be slightly convex to overcome the nonlinearity of π in (19), (20). For example, (19) and (20) hold in the odds model (16).

Auctions

An item, worth $\$V$ to you, is for sale by auction. How much should you bid? Your strategy depends upon the precise mechanics of the auction. In an English auction, people make bids of increasing size until someone has bid an amount that no one cares to exceed. The highest bidder pays his bid price and collects the object. Your strategy should be to bid \$1 higher than the current bid until such a time as (i) you win the object or (ii) you withdraw from the bidding because to continue would require a bid of \$$V$ or more. (See Engelbrecht-Wiggans [1980] for a detailed survey of auctions and bidding.)

In a Dutch auction, the seller starts the proceedings by offering the object for sale at some high price and gradually reduces the price until someone signals that he is prepared to pay that price.

In a sealed bid auction, each bidder writes down a bid and seals it in an envelope. All envelopes are opened simultaneously and the highest bidder pays his bid price and gets the object.

Finally, a philatelist's auction is similar to a sealed bid auction except that the highest bidder, who still gets the object, pays only the bid price of the second highest bidder. It is possible to show that your optimal bid is \$$V$ in a philatelist's auction. Hence, the only information you need in order to bid optimally in either an English or philatelist's auction is the value to you of the auctioned item. If everyone bids optimally, both systems will result in the item being sold to the person who values it the most, at a price equal to the value assigned to it by the person who values it second most.

The Dutch and sealed bid auctions are also equivalent to each other both with respect to informational needs, strategy, and outcome. One should estimate the proability of winning with any given bid B, call this function $p(B)$, and bid the amount that maximizes the quantity $(V - B)p(B)$ which represents the expected value from bidding \$$B$. Assuming $p(B)$ is differentiable and increasing in B, the optimal bid B^* is the solution to the equation

$$B = V - p(B)/p'(B). \qquad (21)$$

However, if we account for disappointment, the optimal strategy in a sealed bid auction is to bid B to maximize

$$(V - B)\pi(p(B)).$$

The optimal bid (called this B^o to differentiate it from B^*, the solution to (21)) is now given by solving the equation

$$B = V - \pi(p(B))/p'(B)\pi'(p(B)). \qquad (22)$$

Even though these equations are implicit in B, it is clear that the optimal bid has been reduced because of disappointment if the right-hand side of (22) is less than that of (21), that is, if the optimal bid B^* from (21) satisfies

$$\pi'(p(B^*)) < \pi(p(B^*))/p(B^*).$$

This equation will always be satisfied by the odds model and will be satisfied by the linear model if $(d - e)p(B^*) < ½$. In the presence of disappointment, low probability high payoff gambles become relatively more attractive so that the optimal bid is reduced.

Equation 22 would also give the correct strategy for a Dutch auction, except for one key distinction. A sealed bid auction is resolved instantaneously (in one stage), but a Dutch auction is resolved in multiple stages as the price is gradually lowered. For example, suppose the asking price has been reduced to $V^*(>B^\circ)$. Your expectations of successfully bidding for the object have increased because you have new information that no one was prepared to bid more than V^*. Your probability of winning with a bid of $B(\leq V^*)$ is now $p(B)/p(V^*)$, where we assume $p(V^*) < 1$. Now your optimal bid has changed to the solution of

$$B = V - \frac{\pi(p(B)/p(V^*))p(V^*)}{p'(B)\pi'(p(B)/p(V^*))}. \qquad (23)$$

(Of course, if all the other bidders suffer from disappointment, their bids will be affected too, thus changing $p(B)$. We will ignore this phenomenon.)

What bid will you actually make? Only when V^*, the asking price, is equal to B^{**}, the solution of (23), will you actually make a bid. Hence, the price at which you will take the object (if the bid price ever becomes that low) is found by solving the following equation for B:

$$B = V - \pi(1)p(B)/p'(B)\pi'(1)$$
$$= V - (1/\pi'(1))\cdot(p(B)/p'(B)). \qquad (24)$$

A comparison of (21) and (24) shows that the effect of disappointment is to increase what you are prepared to pay in a Dutch auction if $\pi'(1) > 1$. It is clear that as the asking price drops close to the price you were originally prepared to pay, the potential for disappointment increases and causes you to raise your bid to eliminate the possibility of losing. This response is well understood by expert negotiators. As you (the naive negotiator we well suppose) finally realize that a settlement is close, your expectations rise. At this point your opponent (an expert negotiator) suddenly backs off from the agreement. Your distress at the impending disappointment causes you to make concessions you would not otherwise have made.

Apart from financial implications, which auction method is preferred from a psychological point of view? The sealed bid and philatelist's auctions break both good and bad news suddenly. The Dutch auction breaks bad news suddenly—someone else bids and you are out—but breaks good news slowly; as the asking price descends you gradually realize you are going to win.

The English auction breaks bad news slowly. As the asking price goes up, you slowly get the message that you are not going to win. If you do win, this news comes to you somewhat slowly as others bidders drop out, but the probability of winning was likely to be no better than half at the end since at least one other person must have been bidding. Clearly, the English auction is kinder on the psyche than the other three. Even better might be a blind English auction in which only the auctioneer is aware of who is making the bids. This procedure would leave you in doubt right up to the end about the number of people left in the bidding and, therefore, presumably, make the victory more sweet.

5. SUMMARY AND CONCLUSIONS

For many decisions it is apparent that any quantitative analysis must account for the various conflicting objectives of the decision maker. Psychological satisfaction, as opposed to satisfaction derived from consumption, is an appropriate objective that should be included in any decision analysis if the decision maker regards it as a criterion for decision. It can be argued that doing so lowers short term financial efficiency, but taking weekday vacations in the mountains also does so. Indeed the benefits may be similar; alleviation of stress may have the intangible payoff of improving the general efficiency of the decision maker.

In particular, a consumer may wish to spend some of those dollars in avoiding disappointment, an aspect of risk aversion that does not seem to be reflected by a utility function over dollar assets alone. This paper does not suggest that people ought to make financial tradeoffs to avoid disappointment, nor does it assert (though I believe it to be true) that people do so. As is the case with normative analyses, it merely indicates the behavior that is the logical result of such an objective. Although the implications of this analysis are, in a number of ways, consistent with behavior observed in laboratory experiments, it would be surprising to hear that subjects become sufficiently involved with their hypothetical choices to make psychological effects primary motivators of their selections. It may be that the psychological impacts of a decision are generated by the same thought process used in making a decision, namely that the value of an outcome is judged relative to various reference points such as status quo, foregone assets, and prior expectations.

Disappointment, and related concepts such as regret, have important implications for the study of decision making under uncertainty. Although the axioms of von Neumann and Morgenstern are the cornerstones of decision analysis, they cannot be expected to hold if preference has not been calculated over all attributes of interest to the decision maker. While it has taken a study of descriptive behavior to force recognition of the importance of psychological impacts to the decision maker, it is not our intent to revise the normative theory continually until it matches empirical evidence. Such an approach would make decision analysis useful as a predictive tool but of less value as a prescriptive tool.

We observed that decision makers often agree with the logic of decision analysis but feel uncomfortable at an intuitive level with its implications. Far from encouraging departure from traditional economic analysis, this paper may satisfy decision makers that what is currently omitted from expected utility analysis *deserves* to be omitted and that a formal analysis may be exactly what is needed to prevent a decision maker's intuition from forcing economically inefficient decisions.

REFERENCES

BELL, D. E. 1982. Regret in Decision Making Under Uncertainty. *Opns. Res.* **30**, 961–981.

BELL, D. E. 1983. Risk Premiums for Decision Regret. *Mgmt. Sci.* **29**, 1156–1166.

DYER, J. S., AND R. K. SARIN. 1982. Relative Risk Aversion. *Mgmt. Sci.* **28**, 875–886.

ELLSBERG, D. 1961. Risk, Ambiguity and the Savage Axioms. *Quart. J. Econ.* **75**, 643–669.

ENGELBRECHT-WIGGANS, R. 1980. Auctions and Bidding Models: A Survey. *Mgmt. Sci.* **26**, 119–142.

KAHNEMAN, D., AND A. TVERSKY. 1979. Prospect Theory: An Analysis of Decision Under Risk. *Econometrica* **47**, 263–291.

KEENEY, R. L. 1982. Decision Analysis—An Overview. *Opns. Res.* **30**, 803–838.

KEENEY, R. L., AND H. RAIFFA. 1976. *Decisions with Multiple Objectives*. John Wiley & Sons, New York.

LOOMES, G., AND R. SUGDEN. 1982. Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty. *Econ. J.* **92**, 805–824.

MACHINA, M. J. 1981. "Rational" Decision Making Versus "Rational" Decision Modelling. *J. Math. Psychol.* **24**, 163–175.

MACCRIMMON, K. R., AND D. A. WEHRUNG. 1983. *Managing Risk*. Department of Commerce, University of British Columbia, Vancouver (unpublished manuscript).

PRATT, J. W. 1964. Risk Aversion in the Small and in the Large. *Econometrica* **32**, 122–136.

PRESTON, M. G., AND P. BARATTA. 1948. An Experimental Study of the Auction-Value of an Uncertain Outcome. *Am. J. Psychol.* **61**, 183–193.

RAIFFA, H. 1968. *Decision Analysis*. Addison-Wesley, Reading, Mass.

SCHELLING, T. 1983. The Mind as a Consuming Organ. John F. Kennedy School of Government, Harvard University, Cambridge, Mass. (unpublished manuscript).

STONE, B. K. 1973. A General Class of Three-Parameter Risk Measures. *J. Finance* **28**, 675–685.

WEHRUNG, D. A., J. F. BASSLER, K. R. MACCRIMMON AND W. T. STANBURG. 1978. Multiple Criteria Dominance Models: An Empirical Study of Investment Preferences. In *Multiple Criteria Problem Solving*, pp. 494–508, S. Zionts (ed.). Springer Verlag, New York.

TAB 12

8.1.9 Potomac Mischaracterized Manitoba Hydro's extrapolated capacity value

In reference to the price of capacity over time, Potomac states, *"With regard to capacity prices, we find no basis for assuming the real price will increase after 2034. For reasons stated above, such prices might even decline."*[105] As noted in Appendix A of Manitoba Hydro's 2013 Electricity Export Price Forecast, the post-2034 real growth rate applied on the reference Capacity price was -0.1%/year resulting in a declining capacity price (in real terms) as Potomac suggests could be a plausible outcome.

8.1.10 Each Consultant Developed their Own Load Growth Assumptions

The Gotham report states, *"Supplemental evidence provided by Manitoba Hydro was in the range of reasonable expectations, but on the high end of the range. The reasons for this include using load forecasts that were not representative of the export region and that did not include the impact of higher prices that would be consistent with the CO2 costs assumed by Manitoba Hydro.*[106]*"* The load forecast that the Gotham report references is outlined on page 6 of their report "*Load Growth in the Export Region*". The Gotham report appears to assume that the indicative macro-level US electric load growth statistics outlined in Chapter 3 of the NFAT filing were provided by Manitoba Hydro to each price forecast consultant as a required input. This is not the case as each price forecast consultant has their own assumption for regional load growth for the specific areas they are modeling. A summary of the consultant level assumptions are available in Appendix C of the 2013 Electricity Export Price forecast that was filed under the NFAT's CSI process.

8.1.11 Carbon Price Embedded within the Export Price Forecast is Reasonable

In preparing their independent forecasts, the export price forecast consultants make their own assessments of a number of pricing factors including but not limited to: fuel price forecasts (coal and natural gas), future load growth forecasts, capital costs and required rates of return, generation retirements and additions, power market rules, future legislative regulations including greenhouse gases and renewable portfolio standard requirements, and characteristics of the existing generation fleet. Therefore, environmental factors including carbon pricing policies are among many factors considered in developing the consensus electricity export price forecast.

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[105] Potomac IEC Report – Page 45 – 3rd paragraph
[106] Gotham report – page 9 – 3rd paragragh

Both Gotham and Potomac have raised concerns about the inclusion of a carbon price in Manitoba Hydro's price forecast. Potomac produced two reference case export price forecasts to consider possible inclusion of carbon pricing. Potomac utilized the reference carbon price forecast of MNP for one forecast and used zero CO2 costs for the second. Potomac considers each scenario equally likely. Manitoba Hydro notes the following regarding the value of carbon embedded in the electricity price forecast:

- First, Manitoba Hydro does not explicitly mandate that a value for carbon must be included within the consultant forecasts. Rather, the value for carbon, if any, is based on the consultants' perspective of the future. In fact [redacted] of the six consultants in the 2013 electricity price forecast have a [redacted] for carbon in their reference case. Therefore the consensus forecast for the reference case is a measured view of the future containing [redacted].
- Second, the annual carbon price embedded in Manitoba Hydro's 2013 electricity price forecast was [redacted] than Myers Norris Penny's (MNP) carbon price forecast.
- Third, the Low Case in the 2013 electricity price forecast and in the 2012 adjusted forecast, [redacted] for carbon throughout the forecast horizon. Therefore it can be noted that Manitoba Hydro does consider a [redacted] through application of its [redacted].
- Fourth, embedding a price for carbon has become a common approach in the Canadian energy industry to capture expected regulation of GHG emissions.[107] Within Manitoba Hydro's export region, Minnesota utilities are explicitly required to develop resources plans that consider a carbon price to ensure the social cost of carbon is being considered in generation project evaluation[108].

MNP attempted to calculate the present value of carbon within the preferred development plan, arriving at $582 million ($2014 dollars), and concluding this was 9% of the total present value revenues of the preferred plan[109]. However, the MNP calculations overestimated the proportion of the revenue due to carbon. MNP improperly extrapolated carbon values beyond 2047 in their analysis, whereas Manitoba Hydro assumed no real

[107] Shadow Carbon Pricing in the Canadian Energy Sector, Sustainable Prosperity, University of Ottawa, March 2013

[108] Minnesota Stat. §216H.06 requires the Commission to establish, by January 1, 2008 and updated annually thereafter, an estimate of the likely range of costs of future carbon dioxide regulation on electricity generation to be used in all electric generation resource acquisition proceedings.

[109] MNP Report A Review of Manitoba Hydro's Macro Environmental Consideration, page 35, and IR PUB –MNP-041 b) which restates the present value to $1,055 at a 5.05% discount rate.

increase in carbon values past 2047. MNP used the carbon value from the 2013 price forecast and compared it to revenues from detailed revenue analysis from SPLASH based on the 2012 adjusted forecast. Further, the MNP methodology assumed a single annual marginal emission value, rather than more detailed modeling which is performed by each of the electricity price forecast consultants. A better estimate of the carbon value was "Carbon prices make up about [redacted] of off-peak electricity prices, about [redacted] of peak opportunity electricity prices, and about [redacted] of long-term dependable electricity prices[110]."

8.1.12 Potomac has Mischaracterized Brattle's Energy Prices

Potomac states that *"[Brattle's] emissions [CO2] and fuel cost assumption along with the high level of coal plant retirements are likely to overstate energy prices."*[111] However, Brattle's carbon price ($15.70 in 2020 to $24 in 2035) is quite similar to Potomac's assumption ($13 in 2021 to $26 in 2035). Brattle's natural gas price ends up being quite similar too, as near as Brattle could determine (there appear to be some differences in the Henry Hub price versus transportation adders, but it appears that these largely offset so that Potomac arrives at a delivered gas price that is similar to Brattle's). Potomac's assumed coal price, based on PRB coal price plus $1.70/MMBtu transportation cost, appears to be significantly above Brattle's coal price, which was based on EIA's delivered regional coal price. The following comparison chart was produced by The Brattle Group to compare the natural gas inputs[112].

[110] LaCapra, Technical Appendix 4 Environmental Issues and Policy, page 4-19.
[111] Potomac IEC report, Page 12
[112] The Brattle Group interpreted Potomac's pricing levels from the charts provided in their report as no data tables were provided.